

Gerardi Meermannii
Specimen calculi fucosum.

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40 Math P. 229



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S. IV.
110.

Mathesis. Analysis finitor. & infinitorum. 347.

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SPECIMEN
CALCULI FLUXIONALIS,

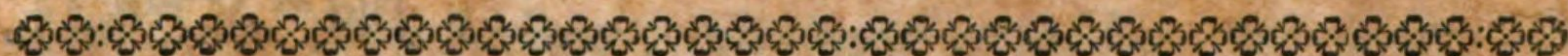
QUO EXHIBETUR

GENERALIS METHODUS

DUARUM PLURIUMVE QUANTITATUM VARIABILIVM
IN SEMET MULTIPLICATARUM FLUXIONES
ET FLUENTES CUJUSCUNQUE ORDINIS
OPE SERIERUM INFINITARUM ADINVENIENDI.

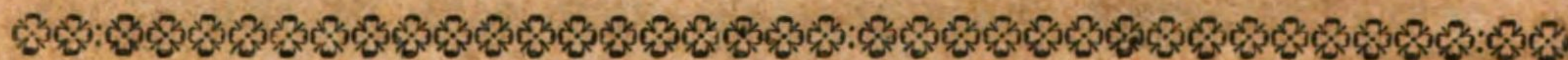
ACCEDUNT ALIA QUÆDAM

MISCELLANEA.



AUCTORE

GERARDO MEERMAN, J^{CTO}.



LUGDUNI BATAVORUM,

Apud DANIELEM GOETVAL.

M D C C X L I I.

2 P E C I M E N
CALCULI FLUXIONUM

GENERALIS METHODUS

INVENIENDI QUANTITATIS
IN ARITHMETICA
ET FLUXIONIBUS
OVS SERIEM INFINITAM

M I S C E L L A N E A

GERARDO MERTEN ANNO 1810



LONDON: A. MILLAR, 178, ST. PAUL'S CHURCH-YARD.
1810.



PRÆFATIO

A D

LECTOREM.



Non leves hominum commerciis suppetias adferre, quin & universæ Matheſeos ſcientiæ plurimum lucis ſænerari Arithmetices ſtudium, neminem arte hac vel leviter tinctum repertum iri exiſtimo, qui inficias ire queat. Hanc proinde neutiquam mirandum eſt, a primis retro temporibus & ipſa quaſi orbis infantia humanum genus utilitatem ejus probe perſpectam habens excoluiſſe: Jam olim etenim Hebræos pulcherrimam artem minime latuiſſe, vel ex ſolis eorum literis numericis, ut alia ſicco pede tranſeam, abunde colligere eſt. Græcos deinceps, apud quos omnes pene ſcientiæ neglectæ antehac & ignorantia tenebris tantum non ſepultæ reviviſcere & diligenter poliri cæperunt, non minimam operam Arithme-

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tica impendisse scripta eorundem, quorum tamen exigua duntaxat pars temporum incuriam, dicam, an injuriam effugere potuit, satis superque testantur: In hoc enim Matheseos genere Pythagoras, Speusippus, Euclides, Archimedes, Nicomachus, alique clari fuere, qui omnes quantum in eo valuerint, nimis notum arbitror, quam ut hoc petitis undequaue testimoniis confirmare necessum fiet: Quin & Analysin (quæ hodie Algebræ nomine venire solet) quondam ipsis, velut Euclidi, & ante hunc Platoni innotuisse inter Eruditos omnes convenit. Narrat etiam Petr. Ramus Lib. I. Algebr. Cap. I. insignem quendam Mathematicum, cujus tamen proprium nomen ignoratur, Algebram suam Syriaca lingua perscriptam ad Alexandrum Magnum misisse, eamque nominasse Almucabalam, hoc est, librum de rebus occultis, eundemque librum hodie adhuc magno apud eruditas illas Orientis nationes esse in pretio, & ab Indis harum artium perstudiosis dici Aljabram, item Alboret: Verum, quum hæc sine auctore scribat Ramus, atque insuper narratiuncula illa exinde, quod Algebram a Gebro aliquo denominatam putaverit, (quod tamen Celeb. Job. Wallisius in Algebra Cap. I. & 2. Tom. II. Opp. Mathem. pag. 2. & 4. solide

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solide refutavit,) originem ducere videatur, an non ea anilibus fabulis accensenda sit, alii judicent: Certe mirari subit, Virum Cl. Job. Bernh. Wideburgium in Disputatione priori de Analyfi Mathematicorum Helmstadii die 2. Novemb. 1715. publico examini submissa §. 10. id ipsum tanquam rem certam & manifestam venditare haud dubitasse: Non est igitur, ut credamus, tam antiquis temporibus integrum aliquod opus super Algebra conscriptum fuisse; maxime, cum Veteres hanc investigandi artem sollicite occultasse ex scriptis eorundem evidens sit: At vero primus, qui Analysin in certam methodum redegit, omnino fuit Diophantus Alexandrinus, de cujus ætate quo magis inter Viros Doctos controverti video, eo facilius Lectorem me excusatum habiturum existimo, si in vera ætate & pulcherrimæ artis inventæ laude pro tanti Viri meritis eidem adserenda paulo latius exspatiari cogar. Et mihi quidem extra omnem dubitationis aleam positum videtur, quod Gregorius Abulpharajius (qui sub exitum sæculi a Nato Christo tertii decimi floruit,) in Historia Dynastiarum pag. 89. Edit. Pocock. nobis scriptum reliquit, Diophantum hunc & Themistium Philosophum sub Juliano Apostata vixisse; quum etenim de Themistii ætate in confes-

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so sit, eam in Juliani tempora incidisse, etiam quoad Diophantum nulla meo equidem judicio ratio suberit, quare fidem optimi Historici sublestam hac in parte habere debeamus: Et quid si dixerim, minime improbabile videri, Abulpharagium id ipsum vel ex vetusto Diophanti Codice Græco, aut, quod verosimilius adhuc, ex ejus Versione Arabica, ad cujus calcem de ipsius ætate nonnihil subjiciebatur, (quod sæpissime factum norunt ii, qui Codices manu exaratos, inprimis Orientales, consuluerunt,) rescire potuisse: Sane libros illos Diophanti de re Arithmetica, (perinde atque complura Veterum Mathematicorum opera,) ex Græca lingua in Arabicam translatos fuisse videor mihi jure concludere posse ex alio Abulpharajii loco pag. 22. ubi Arabem Mohammedem Al-Buzjani, qui Anno Hegiræ 348. sive Christi 959. vixit, Diophanti librum de Algebra interpretatum fuisse scribit: Sive itaque Moham- medes hic Diophantum ipsum simpliciter in Arabicum sermonem verterit, sive etiam Commentarios in eum scripserit, certe Arabibus Auctorem hunc utique cognitum fuisse manifestum existimo; præsertim, quum Abulpharajius dict. pag. 89. Diophanti Librum A, B, quem Algebram vocant, celebrem inquit: Videtur itaque omnis

Ara-

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Arabum in arte hac peritia ex uno fonte Diophantæo manasse; licet enim Job. Wallisius dict. cap. 2. pag. 5. & cap. 13. pag. 68. exinde, quod Arabes alium in recensendis quantitatum dimensionibus ordinem tenuerint, iisdemque, præter supersolidorum adjectionem, alia nomina imposuerint, a contrario argumentari velit, id ipsum tamen mea sententia non magis stringit, quam si quis inde, quod Itali, qui ante duo & amplius sæcula in hac arte clari fuerunt, eam Cossicam appellarint, singulasque potestates novis & ex propria lingua petitis vocabulis insigniverint, concludere vellet, eos itidem scientiam hanc proprio Marte adinvenisse, neque ab Arabibus hausisse; quum tamen ipsi aliud disertis verbis testentur: Et quod ad supersolidorum adjectionem aliumque potestatum ordinem adtinet, ea in parte Diophanteam methodum emendasse dici possunt, & debent, quod etiam Lazar. Schonerum animadvertisse video Lib. de Numeris Figuratis Cap. IV. §. 3. & 4. Cæterum Diophantus egregiæ illius artis inventionem & subtilissimam quæstionum enodatione hanc laudem consecutus fuit, ut non modo præstantissima quævis elogia eidem adscribi videamus, veluti quod a Joanne Patriarcha Hierosolymitano (qui teste Job. Alb. Fabricio in Biblioth. Græc. Tom. VIII. pag.

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pag. 775. *post medium sæculi decimi vixit,*) in vita Joh. Damasceni Cap. II. § II. *apud Henschenium in Actis Sanctorum Tom. II. Maji pag. 113. in fin. & pag. 725. cum ipso Pythagora comparetur;* (*quatenus, ut ego intelligo, sicut Pythagoras artis Arithmeticæ, videl. in certum ordinem redactæ, ita Diophantus Analyticæ primus inventor habetur,*) *verum etiam, quod varios non modo inter Arabes, ut jam dictum, sed & Græcos nactus fuerit Interpretes: Primum etenim celebris illa Hypatia Theonis Alexandrini filia, quam sub Arcadio ac Honorio Impp. floruisse & A. 415. infelicissima morte periisse inter omnes constat, Suida teste in Auctorem hunc Commentaria scripsit: Deinde post novem circiter sæculorum intervallum duos priores libros (de reliquis enim mihi nondum liquet,) interpretatus fuit Monachus Maximus Planudes; illi etenim adscribenda esse, quæ a Guil. Xylandro primo Diophanti editore A. 1575. in lucem protracta sunt, Scholia non jam verosimile at certo certius est; quum hæc illi tribuantur in vetustis quibusdam Codicibus, quos Bibliotheca Regia Parisiensis, Bodlejana, aliæque adservant, teste Bernh. de Montfaucon in Bibliotheca Bibliothecarum MStorum; nihilque refert, quod exemplar illud, quo Xylander*
usus

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usus fuit, Planudis nomen in fronte haud gesserit; quum ὁ
 Ὑποφωρίᾳ illa κατ' Ἰνδὸς, ἡ λεγομένη μεγάλη, quam
 praedicti Monachi esse constat, Anonymo tribuatur in Libro
 MS. Archivii Basilicae S. Petri, si fides Montfauconii
 Catalogo Tom. I. pag. 187. Sane adnotationes illas aucto-
 rem Hypatiam haud agnoscere non modo ex Indica multipli-
 candi ratione, de qua loquitur ad Lib. I. Definit. 9.
 sed etiam ex Algebrae vocabulo (quo in Schol. ad Lib. I.
 Quaest. 24. utitur,) antiquis hisce temporibus inaudito,
 si id observasset, docere potuisset Xylander. Et ex hisce
 quidem plus satis liquere arbitror, Graecis pulcherrimae hu-
 jus artis inventionem, eosque inter ipsi Diophanto (quippe
 qui in Praefat. Lib. I. Arithmet. eam ignotam haecenus
 fuisse scribit,) attribuendam. Ab illis autem ea deinceps
 ad Indos transiit; unde ὁ Planudes ad dict. Lib. I. Dio-
 ph. Definit. 9. Indicam multiplicandi rationem (per signa
 + ὁ -) inversum Graecanici moris ordinem sequi scribit:
 Hac vero Analysis ut promptius ac expeditius uti possent,
 figuras quasdam literarum Alphabeticarum vice in nume-
 rando adhibendas ipsi excogitarunt: Falli enim omnino
 existimo Contr. Dasypodium Lib. I. Instit. Mathem. ὁ
 Petr. Dan. Huetium in Demonstrat. Evangel. Prop.
 IV. qui eas ex corruptis Graecorum literis efformatas fuisse
 exist-

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*existimant, multoque magis Marc. Zuer. Boxbornium in
sijn Antwoord op de Vraaghen, hem voorgestelt over
de Bediedinge van de Afgodinne Nehalennia pag. 36. &
seqq. Edit. Lugd. Bat. 1647. nec non in Originibus Gal-
licis Cap. 8. p. 106. putantem, Indos illas ex Scytharum
Alphabeto mutuasse: Sane numeros hos literis Alphabeticis
minime correspondere, vel ex solo caractere Ɔ, qui in nu-
mericis Arabum figuris quintum, at, si convertatur, in
Alphabeto secundum ordinem obtinet, evinci posse autumo:
Et si, prout Abenragel in Præfat. ad Astronom. testatur,
Brachmanes Indiæ Sapientes numeros ex figura circuli secti
adinvenierint, eos ad literas aliarum gentium Alphabeticas
attendisse, qua ratione defendi queat, non sum, qui vi-
deam. Fuerunt etiam, qui ulterius adhuc progressi sunt,
quibusque hæc sedit sententia, ipsos Græcos & Latinos his
ciphris antehac usos, quos inter & Isaacus Vossius nomen
suum professus fuit in Commentar. ad Pompon. Melæ
Lib I. Geograph. Cap. 12. qui existimabat, se id MS.
Boethii antiquo, item Notis Senecæ ac Tyronis comprobare
posse, camque sententiam Vir Cl. Job. Frid. Weidlerus
in elegantissima Dissertatione de Characteribus numero-
rum vulgaribus & eorum ætatibus Witenbergæ die 18.
Julii 1727. publice defensa propugnavit, provocans ad*

Codi-

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Codicem Boethii, quem in §. 9. sæculo minimum octavo aut nono exaratum fuisse scribit, nec non ad primam ejusdem Auctoris Editionem Venet. 1492. in quibus vulgares numeri a nostris parum recedentes exstabant: At quum ipsum Cod. MS. haud inspexerim, nescius sum, an non de ejus antiquitate forte quæstio moveri posset, de qua etiam dubitare video Job. Christoph. Heilbronnerum in Historia Matheseos universæ hoc ipso Anno edita Lib. IV. Cap. 1. §. 14. p. 741. Minus adhuc probabilis videtur sententia Steph. le Moync in Not. ad varia Sacra Tom. II. p. 799. & seqq. qui ciphras non ex India ad Arabes pervenisse, sed pure Arabicas & ex eorum literis Alphabeticis confectas esse existimabat; maxime quum non modo exteri, velut Job. de Sacro-Bosco & Maximus Planudes, sed & ipsi Arabes (quos inventionis laudem sibi cæteroquin abjudicatueros fuisse quis credat?) Indis hoc inventum attribuant, ut sunt Abenragel cit. loc. & Alsépadi Scholiastes Togræi. Vid. Athanas. Kircher. in Arithmolog. Cap. 4. ut & Job. Wallis Tom. I. Opp. Mathem. p. 48. ac seqq. & Tom. II. p. 9. ac seqq. quibus addere licet, in Bibliotheca nostra Lugduno-Batava exstare Codicem MS. ex Persica lingua in Arabicam translatum super Computo Indico sive Arithmetica auctore Ali Ben Ahmed Nesewensi, in quo eadem figuræ numericæ, quibus Arabes & Planudes

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usi fuere, conspiciuntur. Hæ autem numerorum notæ una cum ipsa arte Analytica ab Indis ad Arabes pervenere, qui scientiæ huic vernacula lingua Algebræ nomen indiderunt: Apud hos vero illa (perinde atque reliquæ Matheseos partes,) non leve cepit incrementum; ut enim de supersolidorum additione aliisque minoris momenti taceam, iis inprimis debetur cubicæ æquationis, de qua in Græcorum scriptis (quæ quidem ad nostram memoriam pervenerunt; nam utrum Diophantus forte in aliquo ex deperditis, vel saltem ineditis Libris id ipsum tradiderit, quis divinabit?) nihil reperias, Algebraica resolutio: Id etenim conjicere licet ex Libelli cujusdam Arabici ab Ampliss. Warnerio in Oriente descripti & Bibliothecæ nostræ publicæ relictæ inscriptione, quæ Latine sic sonat. Omar Ben Ibrahim Al' Ghajamæi Algebra cubicarum æquationum, sive de problematum solidorum resolutione: Quod si verum, audiendus profecto non erit Hier. Cardanus in Algebra Cap. 1. qui inventionem hanc Scipioni cuidam Ferreo tribuit. Imo eadem ratione, qua cubicarum æquationum resolutionem Arabes perspectam habuisse ex prædicti Libri titulo colligere est, etiam biquadraticarum solutionem eos haud latuisse satis probabile est; cum & hæ ope æquationum secundi & tertii ordinis resolvi queant, cujus cæteroquin inventi auctor habetur

Raph.

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Raph. Bombellius. Ceterum, sicut Europæi jam dudum ante, videl. ut Wallisius in Algebra Cap. 4. p. 16. & seq. conjicit, circa medium sæculi decimi figuras numericas ab Arabibus acceperant, ita etiam sub exitum ævi, ut videtur, quarti decimi Algebræ scientiam ab iisdem hausserunt, quamque in Italiam circa Annum 1400. primus attulit Leonardus de Pisa, quem de quadratis numeris librum conscripsisse ait, & ex Diophanto multa eam in rem mutuasse nullus dubitat Guil. Xylander in Epist. Dedicat. ad Diophant. Post hunc Pisanum Lucas de Burgo, Mich. Stifelius, Hier. Cardanus, Raph. Bombellius, aliique complures Gerb. Job. Vossio Lib. III. de Natura Artium Cap. 53. ac seq. & Wallisio dict. Op. Cap. 13. memorati clari fuere, qui omnes, etsi laude sua non sint defraudandi, haud multum tamen ad Algebræ, sicut ab Arabibus ad Europæos pervenerat, promotionem contulisse videntur; donec Franciscus Vieta circa finem ætatis sextæ decimæ Logislices (seu Algebræ) speciosæ introductione, qua non modo incognitas, sed & datas quantitates literis designavit, novam huic scientiæ accessionem fecit, simulque artem hanc ad Geometriam adplicuit: Sed omnis lux demum adfulgere cæpit ab illo tempore, quo Thomas Harriot Anglus, qui jam A°. 1621. obiit, primus mortalium docuit,

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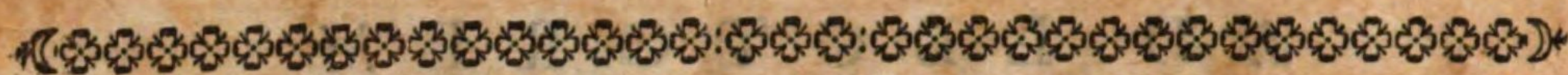
æquationes superiorum ordinum ex simplicioribus in semet ductis constare ; prout ejus Artis Analyticæ Praxis ad æquationes Algebraicas solvendas , quæ Londini A°. 1631. obstetricante Waltero Warnero prodiit , abunde testatur ; ut proinde hujus inventionis laus immerito a quibusdam Renato des Cartes , cujus Geometria , quam verius Algebram dices , demum A°. 1637. prima vice edita fuit , adscribatur ; id quod solide ostendit Cl. Wallisius Tom. II. Opp. Mathem. in Addit. ad Præfat. & pag. 204. & mult. seqq. Ab eo tempore decantari cœpit Arithmetica Infinitorum ab eodem Wallisio A°. 1651. & seq. inventa , quam tamen deinceps , Annis scil. 1665. & 1666. immensum in modum promovit Vir Maximus Isaacus Newtonus introducta nova Fluxionum methodo , quæ postquam cum Nobiliss. Leibnitio communicata esset , ipsi ansam dedit circa An. 1677. Differentialem & Integralem Calculum excogitandi , qua de re videri poterit Recensio Libri , qui inscriptus est , commercium Epistolicum D. Joh. Collins & aliorum de Analyfi promota , ejusdem editioni secundæ A°. 1722. præmissa. Illam deinde fluxionum methodum cum directam tum inversam Viri Celeberrimi Moivreus , Raphsonus , Taylor , Craigijs , Stirlingius , aliique plures editis eam in rem scriptis egregie illustrarunt. Horum vestigia &

ego

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ego premere tentans periclitatus fui, num forte aliquid subsidii intricatæ huic materiæ adferre possem; cui operi et- si vires meæ impares viderentur, nonnihil tamen ab altera parte stimulatum me sensi Senecæ verbis Epist. 64. Agamus bonum patremfamilia. Faciamus ampliora, quæ accepimus. Major ista hereditas a nobis ad posteros trans- eat. Multum adhuc restat operis, multumque restabit, nec ulli nato post mille sæcula præcludetur occasio aliquid adhuc adjiciendi. Proinde, quum alii hucusque fluxioni- bus & fluentibus saltem primi, secundi, tertive ad sum- mum ordinis usi fuerint, ego Cl. Tayloris exemplo genera- lem Regulam ad fluxiones cujuscunque gradus datæ quanti- tatis investigandas præscribere, & exinde novam (ni fal- lor) methodum pro reversione serierum fluxionalium deduce- re conatus fui, fluentes in hisce perpetuo considerans, ut fluxiones negativas. Ipsi vero Opusculo, ut id justa forma lucem adspicere posset, auctarii loco Miscellanea quædam ante tres annos, & quod excurrit, a me conscripta, nunc ve- ro demum ex quadam Observationum Mathematicarum far- ragine selecta subjunxi. Hisce itaque, Lector Benevole, te valere jubeo, utque juveniles hosce conatus æqui bonique consulere velis, enixe rogo. Scrib. Lugduni Batavorum ip- sis Idib. Junii clp Is ccxlii.

TABEL.



TABELLA

*Exhibens casus, qui in Propositione prima & sequentibus
expositi inveniuntur.*

In calculo fluxionali dari potest vel - - - -	ipsa quantitas, & quæri ejusdem vel - - - - -	fluxio, ubi $m > 0$. Vid. <i>Prop. I. Exempl. 1.</i> ad 6.
		fluens, ubi $m < 0$. <i>Prop. I. Exempl. 7.</i> ad 12.
	quantitatis fluxio, ubi $m > 0$, & de- siderari ejusdem vel - - - - -	fluxio, { quæ semper est fluxio reversionis; quoniam ubi $p > 0$, tum $m+p$ multo major 0. <i>Prop. I. Exempl. 15.</i>
		reversio, sive quantitas ipsa; ubi videl. $m+p = 0$. <i>Prop. I. Exempl. 13.</i>
	fluens, ubi $p < 0$, eaque est vel - - -	fluxio reversionis, si $m+p > 0$. <i>Prop. I. Exempl. 16.</i>
		fluens reversionis, quum $m+p < 0$. Vid. <i>ibid.</i>
	quantitatis fluens, ubi $m < 0$, & in- vestiganda propo- ni ejusdem vel - -	fluens, { quæ semper est fluens reversionis; quoniam ubi $p < 0$, tum $m+p$ multo minor 0. <i>Prop. I. Exempl. 17.</i>
		reversio, quando $m+p = 0$. <i>Prop. I. Exem- pl. 14.</i>
	fluxio, ubi $p > 0$, eaque est vel - - -	fluxio reversionis, ubi $m+p > 0$. <i>Prop. I. Exempl. 18.</i>
		fluens reversionis, si $m+p < 0$. Vid. <i>ibid.</i>

SPECI-

S P E C I M E N CALCULI FLUXIONALIS.

P R O P O S I T I O I.

*Quantitatis bimembris, sive fluxionibus sive fluentibus quibuscun-
que involutæ, tam decrementum quam incrementum
cujuslibet ordinis invenire.*



It R quantitas indeterminata: Notum est, R exprimere illius quantitatis decrementum seu fluxionem primam, $\ddot{R}$ secundam, $\ddot{\ddot{R}}$ tertiam, $\ddot{\ddot{\ddot{R}}}$ quartam, & sic deinceps: Nos vero, partim confusionis evitandæ, partim brevitatis ergo pro $\dot{R}$, $\ddot{R}$, $\ddot{\ddot{R}}$, $\ddot{\ddot{\ddot{R}}}$, &c. substituemus $\overset{1}{R}$, $\overset{2}{R}$, $\overset{3}{R}$, $\overset{4}{R}$, &c. Perinde, atque pro fluentibus sive incrementis primi, secundi, tertii, quarti, & ulteriorum ordinum, quæ alii hoc pacto $\underset{1}{R}$, $\underset{2}{R}$, $\underset{3}{R}$, $\underset{4}{R}$, &c. alii vero $\overset{1}{R}$, $\overset{2}{R}$, $\overset{3}{R}$, $\overset{4}{R}$, &c. designant, brevius scribi posset $\overset{1}{R}$, $\overset{2}{R}$, $\overset{3}{R}$, $\overset{4}{R}$, &c. Verum, quia, ut recte animadvertit Vir Cl. Brook Taylor in *Introduct. ad Method. Increment.* lineolæ hæ, quæ genus fluentium exprimunt, super ipsas quantitates, vel puncta sub quantitatibus, quarum incrementa desiderantur, collocata vim habent punctorum negativorum in indicibus fluxionum, hinc pro $\overset{1}{R}$, $\overset{2}{R}$, $\overset{3}{R}$, $\overset{4}{R}$, &c. reponere malumus $\overset{-1}{R}$, $\overset{-2}{R}$, $\overset{-3}{R}$, $\overset{-4}{R}$, &c. quum, si rem accurate introspeciamus, fluens qualiscunque, puta ordinis (n), tan-

A

tun-

tundem valeat, atque fluxio ordinis $(-n)$. Quare generaliter

$\overset{a}{R}$ exprimet fluxionem & fluentem ordinis (a) dictæ quantitatis R : Si enim a nihilum excedat, tum fluxio hac ratione designabitur; quod si nihilo adæquetur, tum pura est quantitas R , id est, neque fluxionibus neque fluentibus involuta; si vero nihilo minor sit, tum $\overset{a}{R}$ denotabit fluentem generis (a) dictæ quantitatis R .

Proponatur jam bimembrum fluxionale, hoc est, duæ quantitates variabiles (sive quæ fluxiones vel fluentes recipere possunt,) in semet multiplicatæ, puta $\overset{a}{R} \times \overset{b}{S}$, ejusque bimembri desideretur sive decrementum sive incrementum ordinis (m) ,

id est, quærat^r $\overset{a}{R} \overset{b}{S}$ in serie infinita. Dico $\overset{a}{R} \overset{b}{S}$ æquale esse

$$\overset{a}{R} \overset{b+m}{S} + \frac{m}{1} \overset{a+1}{R} \overset{b+m-1}{S} + \frac{m}{1} \times \frac{m-1}{2} \overset{a+2}{R} \overset{b+m-2}{S} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \overset{a+3}{R} \overset{b+m-3}{S} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \overset{a+4}{R} \overset{b+m-4}{S} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \times \frac{m-4}{5} \overset{a+5}{R} \overset{b+m-5}{S} + \&c.$$

Vel etiam, si ab

S seriem ordiri velimus, $\overset{b}{S} \overset{a}{R}$ æquabitur $\overset{b}{S} \overset{a+m}{R} + \frac{m}{1} \overset{b+1}{S} \overset{a+m-1}{R} + \frac{m}{1} \times \frac{m-1}{2} \overset{b+2}{S} \overset{a+m-2}{R} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \overset{b+3}{S} \overset{a+m-3}{R} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \overset{b+4}{S} \overset{a+m-4}{R} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \times \frac{m-4}{5} \overset{b+5}{S} \overset{a+m-5}{R} + \&c.$

Legem progressionis in hisce seriebus quod spectat, ea in omnium incurrit oculos: Coëfficientes enim numeri iidem plane sunt cum illis, quos in serie Magni Newtoni pro binomio ad potestatem generis (m) evehendo cernere licet, suntque numeri pyramidales ordinis $(-m-3)$. In priori serie genus varietatis (hoc est decrementi vel incrementi) ipsius R incipit ab a , & deinde in quovis termino unitate successive augetur; at genus fluxionale ipsius S , quod a $b+m$ incipit, in quovis termino unitate minuitur: Ast in altera serie contrarium plane obtinet; ibi etenim index fluxionis vel fluentis ipsius S , qui cum b incipit, continuo monade adcrefcit, dum index quantitatis R , qui ab $a+m$ incipit, monade decrefcit; ita ut in utraque serie summa indicum fluxionalium cujusvis termini semper sit $a+b+m$.

EXEM.

E X E M P L U M I.

QUærat fluxio prima quantitatis $x y$. Satis superque constat, eam æqualem esse $x \dot{y} + \dot{x} y$; at, qua ratione id ipsum ex hisce seriebus sequatur, experiamur: Igitur in prima serie $R = x$, $S = y$, $a = 0$, $b = 0$, $m = 1$: Quare $\dot{x} y$ æquabitur $x \dot{y} + \dot{x} y$; reliqui enim termini, quoniam multiplicati sunt cum $m - 1$, quod in hoc casu nihilo æquale est, evanescunt; unde ex infinita serie in hac specie, perinde atque in omnibus particularibus casibus, ubi m nihilum superat, id est, ubi memorati binomii $R \dot{S}$ fluxio aliqua capienda est, quantitates finitæ redeunt; quum e contra, si m nihilo minor sit, id est, si ipsius $R \dot{S}$ fluens sive incrementum aliquod desideretur, series in omnibus casibus particularibus infinita remaneat. Si porro ex secunda nostra serie fluxionem ipsius $x y$ investigare velimus, invenietur ea, perinde atque in antecedentibus, $x \dot{y} + \dot{x} y$: Proinde & in hoc casu, & in reliquis omnibus, ubi m est numerus affirmativus, utramvis adhibeas seriem, in eandem plane fluxionem incidet; at, ubi m est negativus numerus, ex utraque generali serie quoad exteriorem formam duæ diversæ specialiores prodibunt, ut ex sequentibus manifestum fiet; (nisi tamen $R = S$, & $a = b$ sint, quo casu una eademque series prodibit tam respectu fluentium, quam fluxionum,) licet & illæ revera inter se æquales sint. Fractum vero numerum ipsum m , (sicuti nec a aut b ,) poni haud posse clarum est, quum sic casus poneretur imaginarius; quamvis tota res examen nihilominus subiret: Jam igitur, ubi datæ quantitatis bimembris fluxiones cujuscunque gradus indagamus, priorem duntaxat seriem adhibitori sumus, ne, si & altera uteremur, idem continuo repetere necessum fiet.

E X E M P L U M II.

Quæritur fluxio secunda ipsius $x y$. Igitur $R = x$, $S = y$, $a = 0$, $b = 0$, $m = 2$. Unde $\overset{2}{x} \overset{0}{y} = \overset{1}{x} \overset{2}{y} + 2 \overset{1}{x} \overset{1}{y} + \overset{2}{x} \overset{0}{y}$; reliqui enim termini ab adjectum multiplicatorem $m-2$ in hac specie nihilo æqualem evanescunt: Idem vero obtinebis, si juxta vulgares operandi regulas quantitaturn $\overset{1}{x} \overset{1}{y} + \overset{1}{x} \overset{1}{y}$, quæ ipsius $x y$ fluxionem primam constituunt, iterum fluxionem capias.

Similiter, si quantitatis $x y$ fluxio tertia desideretur, erit $m = 3$; quare $\overset{3}{x} \overset{0}{y} = \overset{3}{x} \overset{3}{y} + 3 \overset{2}{x} \overset{2}{y} + 3 \overset{2}{x} \overset{1}{y} + \overset{3}{x} \overset{0}{y}$. Eodemque modo $\overset{4}{x} \overset{0}{y} = \overset{4}{x} \overset{4}{y} + 4 \overset{3}{x} \overset{3}{y} + 6 \overset{2}{x} \overset{2}{y} + 4 \overset{3}{x} \overset{1}{y} + \overset{4}{x} \overset{0}{y}$. Item $\overset{5}{x} \overset{0}{y} = \overset{5}{x} \overset{5}{y} + 5 \overset{4}{x} \overset{4}{y} + 10 \overset{3}{x} \overset{3}{y} + 10 \overset{2}{x} \overset{2}{y} + 5 \overset{4}{x} \overset{1}{y} + \overset{5}{x} \overset{0}{y}$. Et generaliter $\overset{m}{x} \overset{0}{y} = \overset{m}{x} \overset{m}{y} + \frac{m}{1} \overset{1}{x} \overset{m-1}{y} + \frac{m}{1} \times \frac{m-1}{2} \overset{2}{x} \overset{m-2}{y} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \overset{3}{x} \overset{m-3}{y} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \overset{4}{x} \overset{m-4}{y} + \&c.$ ubi uncia $\frac{m}{1}$, $\frac{m}{1} \times \frac{m-1}{2}$, &c. ut jam antea dixi, sunt coëfficientes potestatis m ; adeoque, ubi $m = 1$, coëfficientes binomii simplicis; ubi $m = 2$, coëfficientes quadrati; si $m = 3$, coëfficientes cubi, & sic deinceps.

E X E M P L U M III.

Investiganda sit fluxio prima quantitatis $\overset{1}{x} \overset{1}{y}$. Jam in hoc casu $R = x$, $S = y$, $a = 1$, $b = 1$, $m = 1$: Unde $\overset{1}{x} \overset{1}{y} = \overset{1}{x} \overset{2}{y} + \overset{2}{x} \overset{1}{y}$.

Similiter, si ejusdem $\overset{1}{x} \overset{1}{y}$ fluxio secunda quæritur, ubi $m = 2$, habebimus $\overset{1}{x} \overset{3}{y} + 2 \overset{2}{x} \overset{2}{y} + \overset{3}{x} \overset{1}{y}$. Eademque ratione fluxio tertia ipsius $\overset{1}{x} \overset{1}{y}$ æquabitur $\overset{1}{x} \overset{5}{y} + 3 \overset{2}{x} \overset{4}{y} + 3 \overset{3}{x} \overset{3}{y} + \overset{4}{x} \overset{2}{y} + \overset{5}{x} \overset{1}{y}$, & fluxio quarta $\overset{1}{x} \overset{7}{y} + 4 \overset{2}{x} \overset{6}{y} + 6 \overset{3}{x} \overset{5}{y} + 4 \overset{4}{x} \overset{4}{y} + \overset{5}{x} \overset{3}{y} + \overset{6}{x} \overset{2}{y} + \overset{7}{x} \overset{1}{y}$, ac generaliter erit $\overset{m}{x} \overset{1}{y} = \overset{m}{x} \overset{m+1}{y} + \frac{m}{1} \overset{1}{x} \overset{m}{y} + \frac{m}{1} \times \frac{m-1}{2} \overset{2}{x} \overset{m-1}{y} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \overset{3}{x} \overset{m-2}{y} + \&c.$

EXEM-

E X E M P L U M I V.

Desideretur fluxio prima quantitatis $x^3 y^8$: Quare $a = 3$,
 $b = 8$, $m = 1$. Proinde habebimus $x^3 y^8 + x^4 y^8$. Ita etiam,
 ponendo $m = 2$, invenietur dictæ quantitatis $x^3 y^8$ fluxio secun-
 da æqualis $x^3 y^{10} + 2 x^4 y^9 + x^5 y^8$. Et, ubi $m = 3$, fluxio tertia
 $x^3 y^{11} + 3 x^4 y^{10} + 3 x^5 y^9 + x^6 y^8$: Quod si $m = 4$ ponatur, erit flu-
 xio quarta $x^3 y^{12} + 4 x^4 y^{11} + 6 x^5 y^{10} + 4 x^6 y^9 + x^7 y^8$, & generali-
 ter $x^3 y^8 + \frac{m}{1} x^4 y^{m+8} + \frac{m}{1} \times \frac{m-1}{2} x^5 y^{m+6} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} x^6 y^{m+5}$
 $+ \&c.$

E X E M P L U M V.

Exempla hætenus proposita pertinent ad inveniendam cu-
 juscunque ordinis fluxionem quantitatum fluxionibus tan-
 tummodo involutarum: Jam si bimembrum ex fluentibus dum-
 taxat constet, pone $x^{-2} y^{-5}$, ejusque sit inveniendæ fluxio prima,
 erit $a = -2$, $b = -5$, $m = 1$; unde quæsitæ fluxio æquabitur $x^{-2} y^{-4}$
 $+ x^{-1} y^{-5}$. Eadem ratione, posito $m = 2$, dictæ quantitatis $x^{-2} y^{-5}$ flu-
 xio secunda invenietur æqualis $x^{-2} y^{-3} + 2 x^{-1} y^{-4} + x^0 y^{-5}$: Quod si po-
 natur $m = 3$, indagabitur fluxio tertia $= x^{-2} y^{-2} + 3 x^{-1} y^{-3} + 3 x^0 y^{-4}$
 $+ x^1 y^{-5}$. Et universaliter decrementum generis (m) prædictæ
 quantitatis æquabitur $x^{-2} y^{m-5} + \frac{m}{1} x^{-1} y^{m-6} + \frac{m}{1} \times \frac{m-1}{2} x^0 y^{m-7} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3}$
 $x^1 y^{m-8} + \&c.$

E X E M P L U M V I.

Detur porro bimembrum partim fluentes partim fluxiones continens, velut $x^{\overline{-4}} y^{\overline{1}}$, ejusque desideretur fluxio prima, erit $a = -4, b = 1, m = 1$. Ideoque quæsitæ fluxio erit $x^{\overline{-4}} y^{\overline{2}} + x^{\overline{-3}} y^{\overline{1}}$. Posito autem $m = 2$, invenietur ejusdem quantitatis $x^{\overline{-4}} y^{\overline{1}}$ fluxio secunda æqualis $x^{\overline{-4}} y^{\overline{3}} + 2 x^{\overline{-3}} y^{\overline{2}} + x^{\overline{-2}} y^{\overline{1}}$. Quod si ponatur $m = 3$, erit dictæ quantitatis fluxio tertia æqualis $x^{\overline{-4}} y^{\overline{4}} + 3 x^{\overline{-3}} y^{\overline{3}} + 3 x^{\overline{-2}} y^{\overline{2}} + x^{\overline{-1}} y^{\overline{1}}$. Et generaliter $x^{\overline{-4}} y^{\overline{m}} = x^{\overline{-4}} y^{\overline{m+1}} + \frac{m}{1} x^{\overline{-3}} y^{\overline{m}} + \frac{m}{1} \times \frac{m-1}{2} x^{\overline{-2}} y^{\overline{m-1}} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} x^{\overline{-1}} y^{\overline{m-2}} + \&c.$

E X E M P L U M V I I.

Quod si vero non fluxio sed fluens sive incrementum quantitatis alicujus bimembris sive fluxionibus sive fluentibus involutæ, sive etiam puræ desideretur, tum quivis animadvertit, indicem m fore numerum negativum, hoc est nihilo minorem; adeoque tum series etiam in particularibus casibus remanebit infinita: Tum etiam facile videre est, utramque seriem adhiberi posse; licet ambæ revera inter se æquales sint.

Sic si quærat fluxio prima quantitatis $x y$, erit $R = x, S = y, a = 0, b = 0, m = -1$, unde juxta primam seriem erit $x^{\overline{-1}} y^{\overline{-1}} = x^{\overline{-1}} y^{\overline{-2}} + x^{\overline{-2}} y^{\overline{-3}} + x^{\overline{-3}} y^{\overline{-4}} + x^{\overline{-4}} y^{\overline{-5}} + x^{\overline{-5}} y^{\overline{-6}} + \&c.$ & juxta secundam seriem æqualis erit $x^{\overline{-1}} y^{\overline{-1}} - x^{\overline{-2}} y^{\overline{-1}} + x^{\overline{-3}} y^{\overline{-2}} - x^{\overline{-4}} y^{\overline{-3}} + x^{\overline{-5}} y^{\overline{-4}} - x^{\overline{-6}} y^{\overline{-5}} + \&c.$ Si vero juxta vulgatam & a Cl. Taylore *Method. Increment. Prop. 11.* adhibitam methodum series has obtinere velimus, multo labore opus erit: Sic igitur tum operatio instituenda est. Ponatur quæsitum integrale quantitatis $x y$, hoc est $\boxed{xy} = x^{\overline{1}} y^{\overline{1}} + p$: Igitur assumendo fluxiones erit $x y = x^{\overline{1}} y^{\overline{1}} + x^{\overline{1}} y^{\overline{0}} + p$, sive $p = -x^{\overline{1}} y^{\overline{0}}$

$\dot{x} \dot{y}$, unde $p = -\frac{\dot{x} \dot{y}}{x y}$; quare $\boxed{x y} = x y - \frac{\dot{x} \dot{y}}{x y}$. Ponatur jam
 secundo $\boxed{x y} = x y + q$, & fumendo fluxiones erit $\dot{x} \dot{y} = \dot{x} \dot{y} + \dot{x} \dot{y} + \dot{q}$; id est, $\dot{q} = -\dot{x} \dot{y}$, quare $q = -\frac{\dot{x} \dot{y}}{x y}$, ideoque $\boxed{x y} = x y - \frac{\dot{x} \dot{y}}{x y} + \frac{\dot{x} \dot{y}}{x y}$. Deinde tertio fiat $\boxed{x y} = x y + r$, & ca-
 piendo fluxiones erit $\dot{x} \dot{y} = \dot{x} \dot{y} + \dot{x} \dot{y} + \dot{r}$, five $\dot{r} = -\dot{x} \dot{y}$, &
 $r = -\frac{\dot{x} \dot{y}}{x y}$. Idcirco $\boxed{x y} = x y - \frac{\dot{x} \dot{y}}{x y} + \frac{\dot{x} \dot{y}}{x y} - \frac{\dot{x} \dot{y}}{x y}$. Si opera-
 tio hoc pacto continuetur, invenietur $\boxed{x y} = x y - \frac{\dot{x} \dot{y}}{x y} + \frac{\dot{x} \dot{y}}{x y} - \frac{\dot{x} \dot{y}}{x y} + \frac{\dot{x} \dot{y}}{x y} - \frac{\dot{x} \dot{y}}{x y} + \dots$ id quod cum prima nostra serie plane
 congruit. Ad secundam vero eadem ratione obtinendam, po-
 natur $\boxed{x y} = x y + p$, & capiendo fluxiones erit $\dot{x} \dot{y} = \dot{x} \dot{y} + \dot{p}$, hoc est $\dot{p} = -\dot{x} \dot{y}$, & $p = -\frac{\dot{x} \dot{y}}{x y}$, unde $\boxed{x y} = x y - \frac{\dot{x} \dot{y}}{x y}$. Fiat jam secundo $\boxed{x y} = x y + q$, & fumendo fluxio-
 nes invenietur $\dot{x} \dot{y} = \dot{x} \dot{y} + \dot{x} \dot{y} + \dot{q}$, seu $\dot{q} = -\dot{x} \dot{y}$; adeoque
 $q = -\frac{\dot{x} \dot{y}}{x y}$, quare $\boxed{x y} = x y - \frac{\dot{x} \dot{y}}{x y} + \frac{\dot{x} \dot{y}}{x y}$. Deinde tertio po-
 natur $\boxed{x y} = x y + r$, & capiendo fluxiones erit $\dot{x} \dot{y} = \dot{x} \dot{y} + \dot{r}$,
 $\dot{r} = -\dot{x} \dot{y}$, & $r = -\frac{\dot{x} \dot{y}}{x y}$; unde $\boxed{x y} = x y - \frac{\dot{x} \dot{y}}{x y} + \frac{\dot{x} \dot{y}}{x y} - \frac{\dot{x} \dot{y}}{x y}$. Si operationem hoc modo profequi velimus,
 inveniemus $\boxed{x y} = x y - \frac{\dot{x} \dot{y}}{x y} + \frac{\dot{x} \dot{y}}{x y} - \frac{\dot{x} \dot{y}}{x y} + \frac{\dot{x} \dot{y}}{x y} - \frac{\dot{x} \dot{y}}{x y} + \dots$ id
 quod cum secunda nostra serie plane coincidit.

EXEM-

EXEMPLUM VIII.

Quod si investigandum sit incrementum secundum ipsius xy , erit iterum $a=0$, $b=0$, $m=-2$; unde ope primæ seriei invenitur $xy = x^{\overline{-2}}y - 2x^{\overline{-2}}y + 3x^{\overline{-3}}y - 4x^{\overline{-4}}y + 5x^{\overline{-5}}y - 6x^{\overline{-6}}y + \&c.$ & ope secundæ $xy - 2x^{\overline{-2}}y + 3x^{\overline{-3}}y - 4x^{\overline{-4}}y + 5x^{\overline{-5}}y - 6x^{\overline{-6}}y + \&c.$ Si hoc ipsum juxta vulgatam methodum invenire velimus, longe adhuc major, quam in modo præcedenti exemplo, labor adhibendus erit; si quidem hic iterum recurrendum erit ad fluentem primam ipsius xy , quæ superius convenienter primæ nostræ seriei inventa est æqualis $x^{\overline{-1}}y - x^{\overline{-1-2}}y + x^{\overline{-2-3}}y - x^{\overline{-3-4}}y + x^{\overline{-4-5}}y - x^{\overline{-5-6}}y + \&c.$ quare ad obtinendam fluentem secundam quantitatis xy , hoc est ad inveniendum $\boxed{xy}$, singulorum terminorum $x^{\overline{-1}}y$, $x^{\overline{-1-2}}y$, &c. capienda sunt incrementa, hæcque omnia simul addenda; verum, quum supponamus, illa secundum vulgatam & a nobis modo enarratam methodum inveniri posse, aggregatum eorum duntaxat ob oculos ponemus: Videlicet

$$\begin{aligned}
 \boxed{x^{\overline{-1}}y} &= x^{\overline{-2}}y - x^{\overline{-1-3}}y + x^{\overline{-2-4}}y - x^{\overline{-3-5}}y + x^{\overline{-4-6}}y - x^{\overline{-5-7}}y + \&c. \\
 - \boxed{x^{\overline{-1-2}}y} &= -x^{\overline{-1-3}}y + x^{\overline{-2-4}}y - x^{\overline{-3-5}}y + x^{\overline{-4-6}}y - x^{\overline{-5-7}}y + \&c. \\
 + \boxed{x^{\overline{-2-3}}y} &= +x^{\overline{-2-4}}y - x^{\overline{-3-5}}y + x^{\overline{-4-6}}y - x^{\overline{-5-7}}y + \&c. \\
 - \boxed{x^{\overline{-3-4}}y} &= -x^{\overline{-3-5}}y + x^{\overline{-4-6}}y - x^{\overline{-5-7}}y + \&c. \\
 + \boxed{x^{\overline{-4-5}}y} &= +x^{\overline{-4-6}}y - x^{\overline{-5-7}}y + \&c. \\
 - \boxed{x^{\overline{-5-6}}y} &= -x^{\overline{-5-7}}y + \&c. \\
 + \&c. &+ \&c.
 \end{aligned}$$

Summa

$$\boxed{xy} = x^{\overline{-2}}y - 2x^{\overline{-1-3}}y + 3x^{\overline{-2-4}}y - 4x^{\overline{-3-5}}y + 5x^{\overline{-4-6}}y - 6x^{\overline{-5-7}}y + \&c.$$

Id

Id quod iterum cum prima nostra serie in hoc exemplo tradita omni ex parte congruit. Jam ad obtinendum secundum incrementum quantitatis $x y$, quod cum secunda nostra serie conveniat, recurrendum iterum erit ad fluentem primam ipsius $x y$, quam in fine anterioris exempli invenimus hancce, $x y - x y + x y - x y + x y - x y + x y - x y + \&c.$ quorum singulorum terminorum vulgari methodo capiendæ sunt fluentes, eæque inventæ simul addendæ hoc pacto.

$$\begin{array}{rcl}
 \boxed{x y} & = & x y - x y + x y - x y + x y - x y + \&c. \\
 - \boxed{x y} & = & - x y + x y - x y + x y - x y + \&c. \\
 + \boxed{x y} & = & + x y - x y + x y - x y + \&c. \\
 - \boxed{x y} & = & - x y + x y - x y + \&c. \\
 + \boxed{x y} & = & + x y - x y + \&c. \\
 - \boxed{x y} & = & - x y + \&c. \\
 + \&c. & & + \&c.
 \end{array}$$

Summa

$$\boxed{x y} = x y - 2 x y + 3 x y - 4 x y + 5 x y - 6 x y + \&c.$$

Convenienter secundæ seriei in hoc ipso exemplo ex universali nostra deductæ. Vides interim, quantopere laborandum sit ad secundam solummodo fluentem quantitatis alicujus hoc modo adinveniendam. Quanto major itaque devoranda foret molestia, si quis ea methodo decimam e. g. fluentem ipsius $x y$ quærere vellet; id etenim fieri haud posset, nisi ope fluentis nonæ, neque hoc rursus, nisi auxilio octavæ, & sic porro; ut proinde tum omnes fluentes a prima usque ad decimam capiendæ essent, qui sane labor immensus ac toediosus admodum foret; quum, si quis nostram seriem adhibeat, nihil plane intersit, utrum v. g. primum an centesimum incrementum quantitatis bimembris in-

B

vesti-

vestigandum sit, & utrumque eadem facilitate reperiri possit.

Generaliter itaque $x^m y$, prout etiam in *Exemplo 2.* indicavimus, æquabitur $x^m y + \frac{m}{1} x^{m-1} y + \frac{m}{1} \times \frac{m-1}{2} x^{m-2} y + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} x^{m-3} y + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} x^{m-4} y + \&c.$ & juxta secundam seriem $x^m y + \frac{m}{1} x^{m-1} y + \frac{m}{1} \times \frac{m-1}{2} x^{m-2} y + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} x^{m-3} y + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} x^{m-4} y + \&c.$

E X E M P L U M I X.

SI quærat incrementum primum ipsius $x^m y$, erit $a=1, b=0, m=-1$; unde ex serie nostra prima sequitur, quæsitæ fluentis valorem fore hunc, $x^m y - x^{m-1} y + x^{m-2} y - x^{m-3} y + x^{m-4} y - x^{m-5} y + \&c.$ (id quod convenit cum secunda serie Br. Tayloris, quam Vir ille Eruditissimus, deque hac scientia optime meritus in *Tract. de Method. Increment. Prop. 11.* alia, & a nobis *Exempl. 7.* memorata methodo adinvenit,) & ex secunda $x^m y - x^{m-1} y + x^{m-2} y - x^{m-3} y + x^{m-4} y - x^{m-5} y + \&c.$ (quod itidem congruit cum prima serie Tayloriana.) Sic si ponamus $m=-2$, habebimus incrementum secundum ipsius $x^m y$ æquale $x^m y - 2 x^{m-1} y + 3 x^{m-2} y - 4 x^{m-3} y + 5 x^{m-4} y - 6 x^{m-5} y + \&c.$ vel etiam, $x^m y - 2 x^{m-1} y + 3 x^{m-2} y - 4 x^{m-3} y + 5 x^{m-4} y - 6 x^{m-5} y + \&c.$, ubi uncia 1, 2, 3, 4, 5, 6, &c. sunt numeri naturales, & coëfficientes binomii ad potestatem (-2) evecti, (sive unitatis divisæ per binomium potestatis quadraticæ,) ac proinde numeri pyramidales generis (-1) . At, posito $m=-3$, erit quantitatis $x^m y$ incrementum tertium æquale $x^m y - 3 x^{m-1} y + 6 x^{m-2} y - 10 x^{m-3} y + 15 x^{m-4} y - 21 x^{m-5} y + \&c.$ vel etiam $x^m y - 3 x^{m-1} y + 6 x^{m-2} y - 10 x^{m-3} y + 15 x^{m-4} y - 21 x^{m-5} y + \&c.$ ubi uncia 1, 3, 6, 10, 15, 21, &c. sunt numeri pyramidales generis (0) , hoc est triangulares, & coëfficientes unitatis divisæ per binomium altitudinis

dinis cubicæ. Quod si ponas $m = -4$, erit fluens quarta ipsius
 $x y$ æqualis $x y - 4 x y + 10 x y - 20 x y + 35 x y - 56 x y +$
 &c. aut etiam $x y - 4 x y + 10 x y - 20 x y + 35 x y - 56 x y +$
 &c. ubi uncia 1, 4, 10, 20, 35, 56, &c. sunt numeri pyra-
 midales primi generis & coëfficientes potestatis (-4) . Si ve-

ro sit $m = -5$, quantitatis $x y$ incrementum quintum æquabitur
 $x y - 5 x y + 15 x y - 35 x y + 70 x y - 126 x y +$ &c., vel
 etiam $x y - 5 x y + 15 x y - 35 x y + 70 x y - 126 x y +$ &c.
 ubi uncia 1, 5, 15, 35, 70, 126, &c. sunt numeri pyrami-
 dales secundi generis, & coëfficientes potestatis (-5) . In ge-

nere vero $x y = x y + \frac{m}{1} x y + \frac{m \times m-1}{1 \times 2} x y + \frac{m \times m-1 \times m-2}{1 \times 2 \times 3} x y + \frac{m \times m-1 \times m-2 \times m-3}{1 \times 2 \times 3 \times 4} x y +$
 $\frac{m \times m-1 \times m-2 \times m-3 \times m-4}{1 \times 2 \times 3 \times 4 \times 5} x y +$ &c. vel etiam $= x y + \frac{m}{1} x y + \frac{m \times m-1}{1 \times 2} x y + \frac{m \times m-1 \times m-2}{1 \times 2 \times 3} x y +$
 $\frac{m \times m-1 \times m-2 \times m-3}{1 \times 2 \times 3 \times 4} x y + \frac{m \times m-1 \times m-2 \times m-3 \times m-4}{1 \times 2 \times 3 \times 4 \times 5} x y +$ &c. ubi, sicut jam dictum
 sub initium hujus Propositionis, coëfficientes sunt potestatis
 (m) , ac numeri pyramidales generis $(-m-3)$, & si in casu
 particulari pro m adhibeatur numerus quicunque negativus, sig-
 na terminis præfixa alternando mutabuntur, nunc $+$, modo $-$,
 id quod ex modo adlatis exemplis evidens puto.

Clariss. Taylor quoque in *Method. Increment. Prop. 12.* ex-
 hibuit nobis seriem quandam ad inveniendam sive fluxionem sive

fluentem ordinis (n) ipsius $r s$, quam nobis tradit sequentem,

$$r s = \frac{n}{1} r s + \frac{n \times n-1}{1 \times 2} r s + \frac{n \times n-1 \times n-2}{1 \times 2 \times 3} r s + \frac{n \times n-1 \times n-2 \times n-3}{1 \times 2 \times 3 \times 4} r s + \text{&c.}$$

Mirari quis forsitan posset hujus differentiae rationem, cur vi-
 del. in serie Tayloriana index, qui fluentis vel etiam fluxionis
 genus exprimit, in quantitate r ab $n-1$ incipiat, & in reliquis
 terminis continuo unitate accrescat; quum in mea secunda se-
 rie ab $n+1$ seu $m+1$ incipiat, & in subsequenter terminis
 semper unitate decrescat; item, quor coëfficientes in Cl.
 Tayloris serie reperiantur hi, $\frac{n}{1}$, $\frac{n \times n-1}{1 \times 2}$, $\frac{n \times n-1 \times n-2}{1 \times 2 \times 3}$, &c. quum
 in mea serie fuerint, $\frac{n}{1}$, $\frac{n \times n-1}{1 \times 2}$, $\frac{n \times n-1 \times n-2}{1 \times 2 \times 3}$, &c. nec non, quid

causæ sit, quare in Tayloriana serie singulis terminis alternatim diversum signum præpositum sit, nunc $+$, modo $-$; quum in mea universali omnes termini uno eodemque signo, videl. $+$, adfecti sint. Ratio hujus rei haud aliunde petenda est, quam quod numerus n sive m , qui est index ordinis fluxionis vel fluentis, apud Cl. Taylorem est adfirmativus, ubi apud nos est ne-

gativus, & contra: Sic e. g. $x^1, x^2, x^3, \&c.$ juxta Tayloris notationem exprimunt fluentem sive incrementum primum, secundum, tertium, &c. quantitatis x ; juxta nostram autem fluxionem sive decrementum primi, secundi, tertii, & ulteriorum

generum: Ita etiam per $x^{-1}, x^{-2}, x^{-3}, \&c.$ Taylor intelligit fluxionem ordinis primi, secundi, tertii, &c.; nos vero fluentem ipsius x totidem altitudinum: Lubet interim aliquot fluxiones & fluentes ipsius $r s$ juxta Tayloris seriem capere, ut videre possit Lector, eas cum nostris omnimodo congruere: Posito igitur $n = -1$, invenies quantitatis $r s$ fluxionem primam æqualem

$\ddot{r} s + \dot{r} \dot{s}$; reliqui enim termini ob multiplicatorem $n+1$ in hac specie nihilo æqualem evanescunt: Si $n = -2$, habebis fluxionem secundam æqualem $\ddot{r} s + 2 \ddot{r} \dot{s} + \dot{r} \ddot{s}$: Quod si $n = -3$, obtinebis fluxionem tertiam æqualem $\ddot{r} s + 3 \ddot{r} \dot{s} + 3 \ddot{r} \ddot{s} + \dot{r} \ddot{\ddot{s}}$, & sic porro. Quod si ponas $n = 1$, invenies ipsius $r s$ incrementum primum æquale $r s - \dot{r} s + \ddot{r} s - \ddot{\dot{r}} s + \ddot{\ddot{r}} s - \&c.$ Si ponas

$n = 2$, tum fluens secunda æquabitur $\dot{r} s - 2 \ddot{r} \dot{s} + 3 \ddot{\dot{r}} \dot{s} - 4 \ddot{\ddot{r}} \dot{s} + 5 \ddot{\ddot{\dot{r}}} \dot{s} - \&c.$ Posito $n = 3$, erit quantitatis $r s$ incrementum

tertium æquale $\ddot{r} s - 3 \ddot{r} \dot{s} + 6 \ddot{\dot{r}} \dot{s} - 10 \ddot{\ddot{r}} \dot{s} + 15 \ddot{\ddot{\dot{r}}} \dot{s} - \&c.$ & sic deinceps, quæ omnia cum iis, quæ ex secunda nostra serie deduximus, plane congruunt. Quod si Tayloris seriem a quantitate s in-

choares, ex hac generali $\dot{r} s - \frac{n}{1} \ddot{r} \dot{s} + \frac{n}{1} \times \frac{n+1}{2} \ddot{\dot{r}} \dot{s} - \frac{n}{1} \times \frac{n+1}{2} \times \frac{n+2}{3} \ddot{\ddot{r}} \dot{s} + \frac{n}{1} \times \frac{n+1}{2} \times \frac{n+2}{3} \times \frac{n+3}{4} \ddot{\ddot{\dot{r}}} \dot{s} - \&c.$ pro quibuscunque incrementis series obtine-

res,

res, quæ cum illis, quas ex priori nostra derivavimus, eadem omnino forent: Facile interim animadvertis, Cl. Tayloris seriem casui singulari accommodatam esse, videl. ubi apud nos est $a=1$, $b=0$; ne dicam, illam nostra paulo intricatiorem esse, neque tam naturalem progressionis legem, qua series dubio procul commendari merentur, eidem inesse: Interim laudamus maximopere Virum Eruditissimum, qui & primus & solus huc usque, quantum quidem mihi innotuit, tam egregiam Analyseos promotionem adinvenit, &, ut alii Viri Docti hisce incitati similia tentare velint, inprimis exoptamus.

E X E M P L U M X.

It investiganda fluens prima quantitatis $x^2 y^4$, erit $a=2$, $b=4$, $m=-1$; unde quæsitæ fluens æquabitur $x^2 y^4 - x^2 y^3 + x^2 y^2 - x^2 y + x^2 - x^2 y^4 + x^2 y^3 - x^2 y^2 + x^2 y - x^2 + x^2 y^4 - x^2 y^3 + x^2 y^2 - x^2 y + x^2 - x^2 y^4 + x^2 y^3 - x^2 y^2 + x^2 y - x^2 + \&c.$ vel etiam $x^2 y^4 - x^2 y^3 + x^2 y^2 - x^2 y + x^2 - x^2 y^4 + x^2 y^3 - x^2 y^2 + x^2 y - x^2 + x^2 y^4 - x^2 y^3 + x^2 y^2 - x^2 y + x^2 - x^2 y^4 + x^2 y^3 - x^2 y^2 + x^2 y - x^2 + \&c.$ Ita, si ponas $m=-2$, invenies ipsius $x^2 y^4$ fluentem secundam æqualem $x^2 y^4 - 2 x^2 y^3 + 3 x^2 y^2 - 4 x^2 y + 5 x^2 - 6 x^2 y^4 + 7 x^2 y^3 - 8 x^2 y^2 + 9 x^2 y - 10 x^2 + 11 x^2 y^4 - 12 x^2 y^3 + 13 x^2 y^2 - 14 x^2 y + 15 x^2 - 16 x^2 y^4 + 17 x^2 y^3 - 18 x^2 y^2 + 19 x^2 y - 20 x^2 + \&c.$ aut etiam $x^2 y^4 - 2 x^2 y^3 + 3 x^2 y^2 - 4 x^2 y + 5 x^2 - 6 x^2 y^4 + 7 x^2 y^3 - 8 x^2 y^2 + 9 x^2 y - 10 x^2 + 11 x^2 y^4 - 12 x^2 y^3 + 13 x^2 y^2 - 14 x^2 y + 15 x^2 - 16 x^2 y^4 + 17 x^2 y^3 - 18 x^2 y^2 + 19 x^2 y - 20 x^2 + \&c.$ In genere vero $x^2 y^4 = x^2 y^4 + \frac{m}{1} x^2 y^3 + \frac{m}{1} \times \frac{m-1}{2} x^2 y^2 + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} x^2 y + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} x^2 + \&c.$ vel $x^2 y^4 + \frac{m}{1} x^2 y^3 + \frac{m}{1} \times \frac{m-1}{2} x^2 y^2 + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} x^2 y + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} x^2 + \&c.$

E X E M P L U M XI.

Desideretur incrementum quantitatis bimembris fluentibus involutæ, v. g. ipsius $x^{-1} y^{-3}$, erit $a=-1$, $b=-3$; unde, posito $m=-1$, invenietur dictæ quantitatis incrementum primum B_3 aqua-

æquale $x^{\overline{-1-4}}y^{\overline{0-5}} - x^{\overline{-5-1}}y^{\overline{0-6}} + x^{\overline{-6-2}}y^{\overline{0-7}} - x^{\overline{-7-3}}y^{\overline{0-8}} + x^{\overline{-8-4}}y^{\overline{0-9}} - x^{\overline{-9-5}}y^{\overline{0-10}} + \&c.$ aut $x^{\overline{-2-3}}y^{\overline{-3-2}} - x^{\overline{-3-4}}y^{\overline{-2-3}} + x^{\overline{-4-5}}y^{\overline{-1-2}} - x^{\overline{-5-6}}y^{\overline{0-1}} + \&c.$ Quod si ponas $m = -2$, obtinebis ipsius xy incrementum secundum æquale $x^{\overline{-1-7}}y^{\overline{2-8}} - 2x^{\overline{-2-8}}y^{\overline{3-9}} + 3x^{\overline{-3-9}}y^{\overline{4-10}} - 4x^{\overline{-4-10}}y^{\overline{5-11}} + 5x^{\overline{-5-11}}y^{\overline{6-12}} - 6x^{\overline{-6-12}}y^{\overline{7-13}} + \&c.$ vel etiam $x^{\overline{-1-7}}y^{\overline{2-8}} - 2x^{\overline{-2-8}}y^{\overline{3-9}} + 3x^{\overline{-3-9}}y^{\overline{4-10}} - 4x^{\overline{-4-10}}y^{\overline{5-11}} + 5x^{\overline{-5-11}}y^{\overline{6-12}} - 6x^{\overline{-6-12}}y^{\overline{7-13}} + \&c.$ Generaliter autem $xy = x^{\overline{-1-3}}y^{\overline{0-6}} + \frac{m}{1}x^{\overline{0-m-4}}y^{\overline{1-m-5}} + \frac{m}{1} \times \frac{m-1}{2}x^{\overline{1-m-5}}y^{\overline{2-m-6}} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3}x^{\overline{2-m-6}}y^{\overline{3-m-7}} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4}x^{\overline{3-m-7}}y^{\overline{4-m-8}} + \&c.$ aut etiam $x^{\overline{-1-3}}y^{\overline{0-6}} + \frac{m}{1}x^{\overline{0-m-4}}y^{\overline{1-m-5}} + \frac{m}{1} \times \frac{m-1}{2}x^{\overline{1-m-5}}y^{\overline{2-m-6}} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3}x^{\overline{2-m-6}}y^{\overline{3-m-7}} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4}x^{\overline{3-m-7}}y^{\overline{4-m-8}} + \&c.$

E X E M P L U M X I I.

It porro quantitas & fluxiones & fluentes continens, veluti xy , ejusque quærat fluxus prima. In hoc casu $a = -1$, $b = 4$, $m = -1$; unde quæsitum incrementum æquabitur sequenti seriei, $x^{\overline{-1-3}}y^{\overline{0-2}} - x^{\overline{-2-4}}y^{\overline{1-3}} + x^{\overline{-3-5}}y^{\overline{2-4}} - x^{\overline{-4-6}}y^{\overline{3-5}} + x^{\overline{-5-7}}y^{\overline{4-6}} - x^{\overline{-6-8}}y^{\overline{5-7}} + x^{\overline{-7-9}}y^{\overline{6-8}} - x^{\overline{-8-10}}y^{\overline{7-9}} + \&c.$ aut etiam huic, $x^{\overline{-2-4}}y^{\overline{1-3}} - x^{\overline{-3-5}}y^{\overline{2-4}} + x^{\overline{-4-6}}y^{\overline{3-5}} - x^{\overline{-5-7}}y^{\overline{4-6}} + x^{\overline{-6-8}}y^{\overline{5-7}} - x^{\overline{-7-9}}y^{\overline{6-8}} + x^{\overline{-8-10}}y^{\overline{7-9}} - x^{\overline{-9-11}}y^{\overline{8-10}} + \&c.$ Quod si ponas $m = -2$, erit prædictæ quantitatis incrementum secundum æquale $x^{\overline{0-1}}y^{\overline{1-0}} - 2x^{\overline{1-2}}y^{\overline{2-1}} + 3x^{\overline{2-3}}y^{\overline{3-2}} - 4x^{\overline{3-4}}y^{\overline{4-3}} + 5x^{\overline{4-5}}y^{\overline{5-4}} - 6x^{\overline{5-6}}y^{\overline{6-5}} + \&c.$ vel $x^{\overline{0-1}}y^{\overline{1-0}} - 2x^{\overline{1-2}}y^{\overline{2-1}} + 3x^{\overline{2-3}}y^{\overline{3-2}} - 4x^{\overline{3-4}}y^{\overline{4-3}} + 5x^{\overline{4-5}}y^{\overline{5-4}} - 6x^{\overline{5-6}}y^{\overline{6-5}} + \&c.$ Universaliter autem xy æquale est $x^{\overline{-1-m-4}}y^{\overline{0-m-3}} + \frac{m}{1}x^{\overline{0-m-3}}y^{\overline{1-m-2}} + \frac{m}{1} \times \frac{m-1}{2}x^{\overline{1-m-2}}y^{\overline{2-m-1}} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3}x^{\overline{2-m-1}}y^{\overline{3-m-0}} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4}x^{\overline{3-m-0}}y^{\overline{4-m-1}} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \times \frac{m-4}{5}x^{\overline{4-m-1}}y^{\overline{5-m-2}} + \&c.$ aut $x^{\overline{-1-m-4}}y^{\overline{0-m-3}} + \frac{m}{1}x^{\overline{0-m-3}}y^{\overline{1-m-2}} + \frac{m}{1} \times \frac{m-1}{2}x^{\overline{1-m-2}}y^{\overline{2-m-1}} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3}x^{\overline{2-m-1}}y^{\overline{3-m-0}} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4}x^{\overline{3-m-0}}y^{\overline{4-m-1}} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \times \frac{m-4}{5}x^{\overline{4-m-1}}y^{\overline{5-m-2}} + \&c.$

E X E M P L U M X I I I.

Series porro hæ non modo fluxiones & fluentes quantitatum exhibent; verum etiam ad inveniendas ope decrementorum

rum vel incrementorum quantitates ipsas utiles sunt: Ut si detur $x^4 y + 4 x^3 y + 6 x^2 y + 4 x y + x$, & quærat^{ur} ejus fluxionis re^{ver}sio, poterit illa quam facillime ope no^{str}arum serie^{rum} inda^{ga}ri: Patet enim ex singulorum terminorum coëfficientibus, esse hanc fluxionem quartam quantitatis alicujus bi^{mem}bris; quare, ut ipsam quantitatem indagemus, singulorum terminorum fluens quarta capi^{en}da erit, vel ope primæ seriei ad hunc modum.

$$\begin{aligned}
 x^4 y &= x^4 y - 4 x^3 y + 10 x^2 y - 20 x y + 35 x - \&c. \\
 4 x^3 y &= + 4 x^3 y - 16 x^2 y + 40 x y - 80 x + \&c. \\
 6 x^2 y &= + 6 x^2 y - 24 x y + 60 x - \&c. \\
 4 x y &= + 4 x y - 16 x + \&c. \\
 x &= + 1 x - \&c.
 \end{aligned}$$

Summa = $x y$, (quum reliqui termini sese mutuo destruant.)

Vel etiam ope secundæ seriei hoc pacto:

$$\begin{aligned}
 x^4 y &= x^4 y - 4 x^3 y + 10 x^2 y - 20 x y + 35 x - \&c. \\
 4 x^3 y &= + 4 x^3 y - 16 x^2 y + 40 x y - 80 x + \&c. \\
 6 x^2 y &= + 6 x^2 y - 24 x y + 60 x - \&c. \\
 4 x y &= + 4 x y - 16 x + \&c. \\
 x &= + 1 x - \&c.
 \end{aligned}$$

Summa = $x y$, perinde ac modo inventum est.

Quod

Quod si id ipsum minori adhuc negotio adinvenire veli-

mus, sic procedendum erit: Videlicet ipsius $x y$, quod sub
initium hujus Propositionis vidimus æquale esse $x y + \frac{m}{1} x y$
 $+ \frac{m}{1} \times \frac{m-1}{2} x y + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} x y + \&c.$ vel etiam $x y +$
 $\frac{m}{1} x y + \frac{m}{1} \times \frac{m-1}{2} x y + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} x y + \&c.$ capienda
iterum erit sive fluxio sive fluens ordinis (p), & habebimus

$$x y = x y + \frac{m+p}{1} x y + \frac{m+p}{1} \times \frac{m+p-1}{2} x y + \frac{m+p}{1} \times \frac{m+p-1}{2} \times \frac{m+p-2}{3} x y + \&c.$$

$$\text{aut etiam } x y + \frac{m+p}{1} x y + \frac{m+p}{1} \times \frac{m+p-1}{2} x y + \frac{m+p}{1} \times \frac{m+p-1}{2} \times \frac{m+p-2}{3} x y + \&c.$$

Itaque in modo pro-
posito casu singulari erat m ab initio æqualis 4; jam vero assu-
mendo $p = -4$, erit $m+p = 0$, ideoque habebimus duntaxat

$x y$; verum ex comparatione solummodo generalis seriei ter-
mini primi $x y$ cum primo termino datæ fluxionis, qui erat
 $x y$, patet a & b singulos nihilo æquales esse; proinde $x y$ erit
datæ fluxionis reversio, id quod cum modo dictis congruit.

Ita etiam, si daretur $x y + \frac{1}{2} x y - \frac{1}{8} x y + \frac{1}{16} x y - \frac{5}{128} x y +$
 $\&c.$ patet ex coëfficientibus, esse $m = \frac{1}{2}$, adeoque esse hanc
fluxionem imaginariam ordinis ($\frac{1}{2}$); jam ad ejus reversionem
inveniendam poni debet $p = -\frac{1}{2}$, ita ut sit $m+p = 0$, quare ha-

bebimus $x y$, jam comparando iterum $x y$ cum primo termi-
no datæ fluxionis, scil. $x y$, patet $a = 0$, & $b+m = 0$; at, quo-

niam m æqualis est $\frac{1}{2}$, erit $b = -\frac{1}{2}$, ideoque reversio erit $x y$.
Sic videmus, fluxionem & etiam fluentem esse posse imagina-
riam, quamvis singuli termini non sint imaginarie fluxionales;
idque semper evenit, ubi $b+m$, vel, si de secunda serie aga-
tur, ubi $a+m$ est numerus integer vel etiam nihilo æqualis:
Quare ex coëfficientibus datæ fluxionis vel fluentis de ejus na-
tura

tura dijudicandum erit; hi enim si sint numeri fracti, erit fluxio vel fluens imaginaria; at si integri, vera. Voluimus id exemplum, quamvis nullius forte usus, hic loci adferre ad ostendendum, series nostras universales esse, utpote quæ non modo ad veras, sed etiam ad imaginarias seu impossibiles fluxiones & fluentes adplicari queunt.

E X E M P L U M X I V.

Detur fluens qualiscunque quantitatis bimembris, ejusque desideretur reversio; ita e. g. detur $x^{\frac{1}{3}}y^{\frac{2}{4}} + 6x^{\frac{3}{5}}y^{\frac{4}{6}} - 10x^{\frac{4}{6}}y^{\frac{5}{7}} + 15x^{\frac{5}{7}}y^{\frac{6}{8}} - \&c.$ vel $x^{\frac{1}{3}}y^{\frac{2}{4}} - 3x^{\frac{2}{4}}y^{\frac{3}{5}} + 6x^{\frac{3}{5}}y^{\frac{4}{6}} - 10x^{\frac{4}{6}}y^{\frac{5}{7}} + 15x^{\frac{5}{7}}y^{\frac{6}{8}} - \&c.$ Apparet satis ex coëfficientibus, esse hanc fluentem tertiam quantitatis alicujus; quare ad inveniendam ipsam quantitatem capienda est harum serierum fluxio tertia hoc pacto:

$$\begin{aligned}
 x^{\frac{3}{1-3}}y^{\frac{1}{2-1}} &= x^{\frac{3}{1-3}}y^{\frac{1}{2-1}} + 3x^{\frac{2-1}{2-1}}y^{\frac{3-2}{3-2}} + 3x^{\frac{3-2}{3-2}}y^{\frac{4-3}{4-3}} + 1x^{\frac{4-3}{4-3}}y^{\frac{5-4}{5-4}} \\
 - 3x^{\frac{3}{2-4}}y^{\frac{2-1}{2-1}} &= - 3x^{\frac{3}{2-4}}y^{\frac{2-1}{2-1}} - 9x^{\frac{3-2}{3-2}}y^{\frac{4-3}{4-3}} - 9x^{\frac{4-3}{4-3}}y^{\frac{5-4}{5-4}} - 3x^{\frac{5-4}{5-4}}y^{\frac{6-5}{6-5}} \\
 + 6x^{\frac{3}{3-5}}y^{\frac{3-2}{3-2}} &= + 6x^{\frac{3}{3-5}}y^{\frac{3-2}{3-2}} + 18x^{\frac{4-3}{4-3}}y^{\frac{5-4}{5-4}} + 18x^{\frac{5-4}{5-4}}y^{\frac{6-5}{6-5}} + \&c. \\
 - 10x^{\frac{3}{4-6}}y^{\frac{4-3}{4-3}} &= - 10x^{\frac{3}{4-6}}y^{\frac{4-3}{4-3}} - 30x^{\frac{5-4}{5-4}}y^{\frac{6-5}{6-5}} - \&c. \\
 + 15x^{\frac{3}{5-7}}y^{\frac{5-4}{5-4}} &= + 15x^{\frac{3}{5-7}}y^{\frac{5-4}{5-4}} + \&c. \\
 - \&c. &= - \&c.
 \end{aligned}$$

$$\text{Summa} = x^{\frac{1}{3}}y^{\frac{2}{4}}.$$

C

Quod

Quod si ope secundæ seriei datæ reversio invenienda sit, operatio instituetur sequentem in modum:

$$\begin{array}{rcl}
 \frac{3}{-2} x y & = & x y + 3 x y + 3 x y + 1 x y \\
 - \frac{3}{-3} x y & = & - 3 x y - 9 x y - 9 x y - 3 x y \\
 + \frac{3}{-4} x y & = & + 6 x y + 18 x y + 18 x y + \&c. \\
 - \frac{3}{-5} x y & = & - 10 x y - 30 x y - \&c. \\
 + \frac{3}{-6} x y & = & + 15 x y + \&c. \\
 - \&c. & & - \&c.
 \end{array}$$

Summa = $x y$, ut in præcedenti casu.

Si id ipsum ope serierum, quas *Exempl. 13.* proposuimus, exsequi velimus, res ultro patebit; nam ab initio erat $m = -3$; igitur ad obtinendam ipsam quantitatem poni debet $p + m = 0$, adeoque $p = 3$, & habebimus pro utraque serie $x y$; jam vero, quoniam primus terminus datæ seriei erat $x y$, qui correspondebat cum $x y$, item $x y$, qui congruebat cum $x y$, at m æqualis est -3 , patet, $a = 1$, & $b = 0$; unde habebimus $x y$, perinde ac modo adinvenimus.

E X E M P L U M X V.

It jam data fluxio qualiscunque quantitatis bimembris, sitque ejusdem fluxionis iterum fluxio cujuslibet ordinis invenienda. Ita detur $x y + 3 x y + 3 x y + x y$, sitque ejus fluxionis secunda fluxio reperienda. Patet jam ex coëfficientibus datæ fluxionis, juxta ea, quæ *Exempl. 13.* docuimus, esse eam

eam ordinis tertii quantitatis $x y$, unde erit $m=3$, jam, quoniam ejus fluxionis rursus fluxio secunda quæritur, erit $p=2$, adeoque $m+p=5$, unde desiderata fluxio erit $x y + 5 x y + 10 x y + 10 x y + 5 x y + x y$, quæ proinde est quinta fluxio ipsius $x y$. Cæterum potest etiam quæsitum obtineri, si singulorum terminorum datæ fluxionis capiamus fluxionem secundam; verum tum calculus paulo prolixior foret, idque adhuc magis, si m esset numerus satis magnus; idque quoniam semper terminorum numero $m+1$ capienda esset fluxio; quare modo enarrata methodo omnia hæc breviori tempore inveniri queunt.

E X E M P L U M X V I.

Proponatur jam fluxio aliqua quantitatis bimembris, sitque ejus fluxionis fluens quædam invenienda. Ita detur $x y + 4 x y + 6 x y + 4 x y + x y$, & quæretur ejus fluens tertia: Novimus jam ex antecedentibus, esse fluxionem datam quarti ordinis ipsius $x y$, unde $m=4$; jam, quoniam dictæ fluxionis fluens tertia desideratur, erit $p=-3$, quare $m+p=1$; unde fluens tertia fluxionis quartæ ipsius $x y$, hoc est fluxio prima quantitatis $x y$ æqualis erit $x y + x y$.

Quod si quæreretur dictæ fluxionis quartæ fluens quarta, jam incideres in ipsam quantitatem $x y$, juxta ea, quæ *Exempl.* 13. vidimus.

At si ulterior fluens investiganda foret, tum inveniretur ipsius $x y$ fluens aliqua: Ita si prædictæ fluxionis fluens sexta invenienda esset, foret $p=-6$, & per consequens $m+p=-2$; unde fluens sexta fluxionis quartæ, hoc est fluens secunda ip-

fius $x y$ tum æqualis erit $x y - 2 x y + 3 x y - 4 x y + 5 x y - 6 x y + \&c.$ vel etiam $x y - 2 x y + 3 x y - 4 x y + 5 x y - 6 x y + \&c.$

E X E M P L U M X V I I.

Detur jam fluens quædam, cujus rursus fluentem aliquam inveniri oportet. Ita detur e. g. $x^{\overline{1}}y - x^{\overline{1-2}}y + x^{\overline{2-3}}y - x^{\overline{3-4}}y + x^{\overline{4-5}}y - \&c.$ vel $x^{\overline{-1}}y - x^{\overline{-2\ 1}}y + x^{\overline{-3\ 2}}y - x^{\overline{-4\ 3}}y + x^{\overline{-5\ 4}}y - \&c.$ (quam jam apparet esse fluentem primam quantitatis xy), & sit datæ fluentis tertia fluens, hoc est ipsius xy quarta fluens invenienda, erit $a=0$, $b=0$, $m=-1$, $p=-3$, adeoque $m+p=-4$; quare habebimus $x^{\overline{-4}}y - 4x^{\overline{-3\ 1}}y + 10x^{\overline{-2\ 6}}y - 20x^{\overline{-1\ 7}}y + 35x^{\overline{0\ 8}}y - \&c.$ aut etiam $x^{\overline{-4}}y - 4x^{\overline{-5\ 1}}y + 10x^{\overline{-6\ 2}}y - 20x^{\overline{-7\ 3}}y + 35x^{\overline{-8\ 4}}y - \&c.$

E X E M P L U M X V I I I.

Proponatur jam fluens aliqua, & desideretur hujus fluxio quædam. Ita detur $x^{\overline{-3}}y - 3x^{\overline{-1\ 4}}y + 6x^{\overline{2\ 5}}y - 10x^{\overline{3\ 6}}y + 15x^{\overline{4\ 7}}y - \&c.$ vel $x^{\overline{-3}}y - 3x^{\overline{-4\ 1}}y + 6x^{\overline{-5\ 2}}y - 10x^{\overline{-6\ 3}}y + 15x^{\overline{-7\ 4}}y - \&c.$ Patet jam ex coefficientibus, esse hanc fluentem tertiam quantitatis xy . Sit jam invenienda datæ fluentis fluxio octava, hoc est ipsius xy fluxio quinta, erit $a=0$, $b=0$, $m=-3$, $p=8$, adeoque $m+p=5$, unde quæsitæ fluxio erit $x^{\overline{5}}y + 5x^{\overline{1\ 4}}y + 10x^{\overline{2\ 3}}y + 10x^{\overline{3\ 2}}y + 5x^{\overline{4\ 1}}y + x^{\overline{5}}y.$

Quod si quæretur fluxio tertia dictæ fluentis tertiæ, foret $m+p=0$, & sic ipsa inveniretur reversio, secundum ea, quæ *Exempl. 14.* vidimus.

At si anterior fluxio invenienda foret, tum incideres in fluentem aliquam ipsius xy . Ita si prædictæ fluentis fluxio secunda quæreretur, foret $p=2$, & per consequens $m+p=-1$; unde fluentis tertiæ fluxio secunda, hoc est quantitatis xy prima fluens æqualis erit $x^{\overline{-1}}y - x^{\overline{1-2}}y + x^{\overline{2-3}}y - x^{\overline{3-4}}y + x^{\overline{4-5}}y - \&c.$ vel etiam $x^{\overline{-1}}y - x^{\overline{-2\ 1}}y + x^{\overline{-3\ 2}}y - x^{\overline{-4\ 3}}y + x^{\overline{-5\ 4}}y - \&c.$

EXEM-

E X E M P L U M X I X .

Q Uod si quis ope seriei nostræ invenire desideret fluxionem
 vel fluentem quantitatis alicujus duarum dimensionum,

e. g. ipsius x , hoc est $x x$, tum erit $R=S$, & $a=b$, ideo-
 que erit ejus decrementum & incrementum cujuscunque ordi-
 nis, $\overset{a}{x} \overset{a+m}{x} + \frac{m}{1} \overset{a+1}{x} \overset{a+m-1}{x} + \frac{m}{1} \times \frac{m-1}{2} \overset{a+2}{x} \overset{a+m-2}{x} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \overset{a+3}{x} \overset{a+m-3}{x} + \&c.$
 Secunda autem series cum hac plane coincidit, non modo quoad
 fluxiones, uti alias; sed etiam quoad fluentes.

E X E M P L U M X X .

A T si ope seriei nostræ invenienda sit fluens prima ipsius
 $2 x x$, erit $R=S=x$, $a=0$, $b=1$, $m=-1$, & proinde
 juxta primam seriem habebimus $2 x x - 2 \overset{1-1}{x} \overset{2-2}{x} + 2 \overset{3-3}{x} \overset{4-4}{x} - 2 \overset{5-5}{x} \overset{6-6}{x} +$
 $2 \overset{7-7}{x} \overset{8-8}{x} - 2 \overset{9-9}{x} \overset{10-10}{x} + \&c.$ & juxta secundam $2 \overset{1-1}{x} \overset{2-2}{x} - 2 \overset{3-3}{x} \overset{4-4}{x} + 2 \overset{5-5}{x} \overset{6-6}{x}$
 $- 2 \overset{7-7}{x} \overset{8-8}{x} + 2 \overset{9-9}{x} \overset{10-10}{x} - \&c.$ & proinde, quoniam hæ duæ series in-
 ter se æquales sunt, erit $2 x x = 4 \overset{1-1}{x} \overset{2-2}{x} - 4 \overset{3-3}{x} \overset{4-4}{x} + 4 \overset{5-5}{x} \overset{6-6}{x} - 4 \overset{7-7}{x} \overset{8-8}{x}$
 $+ 4 \overset{9-9}{x} \overset{10-10}{x} - \&c.$, & dividendo per 2, erit $x x = 2 \overset{1-1}{x} \overset{2-2}{x} - 2 \overset{3-3}{x} \overset{4-4}{x} +$
 $2 \overset{5-5}{x} \overset{6-6}{x} - 2 \overset{7-7}{x} \overset{8-8}{x} + 2 \overset{9-9}{x} \overset{10-10}{x} - \&c.$ Sicque videmus, quantitatem pu-
 ram, id est fluxionibus vel fluentibus non involutam, in seriem
 infinitam fluxionalem resolvi eique adæquari; qua de re si quis
 dubitet, capiat tantummodo ipsius $x x = 2 \overset{1-1}{x} \overset{2-2}{x} - 2 \overset{3-3}{x} \overset{4-4}{x} + 2 \overset{5-5}{x} \overset{6-6}{x}$
 $- 2 \overset{7-7}{x} \overset{8-8}{x} + 2 \overset{9-9}{x} \overset{10-10}{x} - \&c.$ fluxionem primam, & inveniet $2 x x =$
 $2 \overset{1}{x} \overset{2-1}{x} + 2 \overset{2-1}{x} \overset{3-2}{x} - 2 \overset{3-2}{x} \overset{4-3}{x} + 2 \overset{4-3}{x} \overset{5-4}{x} - 2 \overset{5-4}{x} \overset{6-5}{x} + 2 \overset{6-5}{x} \overset{7-6}{x} - 2 \overset{7-6}{x} \overset{8-7}{x} + \&c.$ unde,
 cum hi termini continuo sese mutuo destruant, restabit dun-
 taxat $2 x x = 2 x x$; quare, cum hæ fluxiones sint æquales,

erunt & earum incrementa seu quantitates ipsæ æquales: Jam vero, cum inventum sit, $2x\overset{1-1}{x}-2x\overset{2-2}{x}+2x\overset{3-3}{x}-2x\overset{4-4}{x}+2x\overset{5-5}{x}-$ &c. æquari ipsi $x\overset{1}{x}$, erit juxta priorem seriem fluens prima ipsius $2x\overset{1}{x}$ æqualis $2xx-xx$, hoc est xx , & juxta secundam statim xx , id quod veritati consentaneum esse nemo diffitebitur.

E X E M P L U M X X I.

QUod si quis pro quantitate R vel S adhiberet quantitatem constantem, e. g. n , tum primus duntaxat terminus usui veniet, & reliqui, quoniam quantitas constans fluxiones & fluentes non admittit, evanescent: Ita ipsius $n\overset{a}{x}$, seu $\overset{a}{x}n$ decrementum & incrementum ordinis m convenienter secundæ seriei est $n\overset{a+m}{x}$, (prior etenim series in eo casu locum non habet; quoniam juxta eam quantitas n variabilis foret; quum tamen illa jam constans posita sit,) adeoque, si ipsius $n\overset{1}{x}$ fluxio prima requiratur, erit $a=0$, & $m=1$; unde invenies $n\overset{1}{x}$; item si ipsius $n\overset{2}{x}$ fluens secunda desideretur, erit $a=2$, $m=-2$; unde habebimus nx , & sic de reliquis.





PROPOSITIO II.

Trium quantitatum fluxionalium in semet multiplicatarum, sive decrementum sive incrementum cujuscunque generis invenire.

Sit productum $\dot{R}^a \dot{S}^b \dot{T}^c$ ex tribus quantitibus variabilibus constans, ejusque sit aut fluxio aut etiam fluens qualiscunque, cujus ordo indice m exprimitur, investiganda, dico ipsum

$\dot{R}^a \dot{S}^b \dot{T}^c$ æquale fore sequenti seriei infinitæ:

$$\begin{aligned} & \dot{R}^a \dot{S}^b \dot{T}^{c+m} + \frac{m}{1} \left\{ \begin{matrix} \dot{R}^a \dot{S}^{b+1} \\ \dot{R}^{a+1} \dot{S}^b \end{matrix} \right\} \dot{T}^{c+m-1} + \frac{m}{1} \times \frac{m-1}{2} \left\{ \begin{matrix} \dot{R}^a \dot{S}^{b+2} \\ \dot{R}^{a+1} \dot{S}^{b+1} \\ \dot{R}^{a+2} \dot{S}^b \end{matrix} \right\} \dot{T}^{c+m-2} \\ & + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \left\{ \begin{matrix} \dot{R}^a \dot{S}^{b+3} \\ \dot{R}^{a+1} \dot{S}^{b+2} \\ \dot{R}^{a+2} \dot{S}^{b+1} \\ \dot{R}^{a+3} \dot{S}^b \end{matrix} \right\} \dot{T}^{c+m-3} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \left\{ \begin{matrix} \dot{R}^a \dot{S}^{b+4} \\ \dot{R}^{a+1} \dot{S}^{b+3} \\ \dot{R}^{a+2} \dot{S}^{b+2} \\ \dot{R}^{a+3} \dot{S}^{b+1} \\ \dot{R}^{a+4} \dot{S}^b \end{matrix} \right\} \dot{T}^{c+m-4} \\ & + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \times \frac{m-4}{5} \left\{ \begin{matrix} \dot{R}^a \dot{S}^{b+5} \\ \dot{R}^{a+1} \dot{S}^{b+4} \\ \dot{R}^{a+2} \dot{S}^{b+3} \\ \dot{R}^{a+3} \dot{S}^{b+2} \\ \dot{R}^{a+4} \dot{S}^{b+1} \\ \dot{R}^{a+5} \dot{S}^b \end{matrix} \right\} \dot{T}^{c+m-5} + \&c. \end{aligned}$$

Aut etiam huic:

$\dot{R}^a$

$$\begin{matrix} a & c & b+m \\ \dot{R} & \dot{T} & \dot{S} \end{matrix} + \frac{m}{1} \left\{ \begin{matrix} a & c+1 \\ \dot{R} & \dot{T} \\ a+1 & c \end{matrix} \right\} \begin{matrix} b+m-1 \\ \dot{S} \end{matrix} + \frac{m}{1} \times \frac{m-1}{2} \left\{ \begin{matrix} a & c+2 \\ \dot{R} & \dot{T} \\ a+1 & c+1 \\ a+2 & c \end{matrix} \right\} \begin{matrix} b+m-2 \\ \dot{S} \end{matrix}$$

$$+ \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \left\{ \begin{matrix} a & c+3 \\ \dot{R} & \dot{T} \\ a+1 & c+2 \\ a+2 & c+1 \\ a+3 & c \end{matrix} \right\} \begin{matrix} b+m-3 \\ \dot{S} \end{matrix} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \left\{ \begin{matrix} a & c+4 \\ \dot{R} & \dot{T} \\ a+1 & c+3 \\ a+2 & c+2 \\ a+3 & c+1 \\ a+4 & c \end{matrix} \right\} \begin{matrix} b+m-4 \\ \dot{S} \end{matrix}$$

$$+ \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \times \frac{m-4}{5} \left\{ \begin{matrix} a & c+5 \\ \dot{R} & \dot{T} \\ a+1 & c+4 \\ a+2 & c+3 \\ a+3 & c+2 \\ a+4 & c+1 \\ a+5 & c \end{matrix} \right\} \begin{matrix} b+m-5 \\ \dot{S} \end{matrix} + \&c.$$

Vel denique huic:

$$\begin{matrix} b & c & a+m \\ \dot{S} & \dot{T} & \dot{R} \end{matrix} + \frac{m}{1} \left\{ \begin{matrix} b & c+1 \\ \dot{S} & \dot{T} \\ b+1 & c \end{matrix} \right\} \begin{matrix} a+m-1 \\ \dot{R} \end{matrix} + \frac{m}{1} \times \frac{m-1}{2} \left\{ \begin{matrix} b & c+2 \\ \dot{S} & \dot{T} \\ b+1 & c+1 \\ b+2 & c \end{matrix} \right\} \begin{matrix} a+m-2 \\ \dot{R} \end{matrix}$$

$$+ \frac{m}{1}$$

$$\begin{aligned}
 & + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \left\{ \begin{array}{c} b \\ 1 \dot{S} \dot{T} \\ b+1 \quad c+2 \\ 3 \dot{S} \dot{T} \\ b+2 \quad c+1 \\ 3 \dot{S} \dot{T} \\ b+3 \quad c \\ 1 \dot{S} \dot{T} \end{array} \right\} \dot{R}^{a+m-3} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \left\{ \begin{array}{c} b \\ 1 \dot{S} \dot{T} \\ b+1 \quad c+3 \\ 4 \dot{S} \dot{T} \\ b+2 \quad c+2 \\ 6 \dot{S} \dot{T} \\ b+3 \quad c+1 \\ 4 \dot{S} \dot{T} \\ b+4 \quad c \\ 1 \dot{S} \dot{T} \end{array} \right\} \dot{R}^{a+m-4} \\
 & + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \times \frac{m-4}{5} \left\{ \begin{array}{c} b \\ 1 \dot{S} \dot{T} \\ b+1 \quad c+4 \\ 5 \dot{S} \dot{T} \\ b+2 \quad c+3 \\ 10 \dot{S} \dot{T} \\ b+3 \quad c+2 \\ 10 \dot{S} \dot{T} \\ b+4 \quad c+1 \\ 5 \dot{S} \dot{T} \\ b+5 \quad c \\ 1 \dot{S} \dot{T} \end{array} \right\} \dot{R}^{a+m-5} + \&c.
 \end{aligned}$$

Quod ad ordinem in hisce seriebus observatum adtinet, iis id commune est, quod eosdem habeant coëfficientes, & easdem quoque uncias, eundemque etiam columnarum & singulorum terminorum numerum. In prima columna coëfficiens est monas, in secunda $\frac{m}{1}$, in tertia $\frac{m}{1} \times \frac{m-1}{2}$, in quarta $\frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3}$, & sic porro; perinde, ac circa quantitatem bimembrem sub initium antecedentis Propositionis vidimus. In prima columna terminus est unus, uncia vero nulla conspicitur: In secunda termini adsunt duo; uncia sunt 1, 1, videl. coëfficientes binomii simplicis: In tertia terminos habes tres; uncias 1, 2, 1, quæ sunt coëfficientes binomii potestatis quadraticæ: In columna quarta terminos vides adesse quatuor; uncias autem 1, 3, 3, 1, nempe coëfficientes binomii ad cubicam potestatem evecti; & sic deinceps: Generaliter itaque in qualibet columna, quam litera l jam designo, terminos habes numero l , & uncias potestatis $l-1$. Jam vero, si de indicibus generis fluxionum & fluentium quoad singulos terminos designandis agatur, dico quoad pri-

D

mam

nam seriem in quacunque columna, puta l , singulos terminos in dicta columna repperiundos habituros generalem multiplicatorem, videl. T : Et, quod ad ipsos terminos in quavis columna cum dicta quantitate T multiplicatos adtinet, primus eorum semper erit R S : In reliquis deinde index fluxionalis quantitatis R continuo unitate adcrefcit, & index quantitatis S unitate decrefcit, idque donec perveniatur ad R S , qui proinde semper est postremus columnæ l terminus. In secunda porro serie generalis terminorum in quavis columna multiplicator est S , primusque terminus cum eo multiplicatus est R T , ultimus vero R T . In tertia serie generalis multiplicator in qualibet columna est R , primusque in eadem terminus S T , postremus autem S T : Quare in omnibus tribus seriebus semper & indistincte summa indicum fluxionalium ipsius R S T est $a + b + c + m$. Hæc de lege progressionis in hisce seriebus dicta sunt: Nunc iterum illas ad casus quosdam particulares adplicabimus, tam circa fluxiones, quam fluentes; ubi rursus observandum erit, si numerus m sit affirmativus integer, adeoque si de vera fluxione aliqua investiganda agatur, tres hasce series idem omnino, & quidem semper quantitates numero finitas, producturas; at si numerus m sit negativus, adeoque si incrementum qualecunque desideretur, tres diversas prodituras series, etsi perfecte æquales.

E X E M P L U M I.

Desideretur fluxio quantitatis trimembris, sive hæc fluxiones tantum contineat, sive fluentes tantum, sive etiam utrasque, sive neutras: Ita quærat e. g. fluxio prima ipsius $x y z$: Hoc casu igitur erit $R = x$, $S = y$, $T = z$, $a = b = c = 0$, $m = 1$; unde invenitur dictæ quantitatis $x y z$ fluxio prima æqualis

lis $x y z + x y z + x y z$; ulteriorum etenim columnarum termini ob adjectum multiplicatorem $\frac{m}{1} \times \frac{m-1}{2}$ in hoc casu nihilo æqualem evanescunt. Quod si ponamus $m=2$, indagabimus ipsius $x y z$ decrementum secundi ordinis æquale $x y z + 2 x y z + 2 x y z + x y z + 2 x y z + x y z$, ubi easdem habes uncias, quales invenires, si trinomium, hoc est summam trium quantitatum, ad potestatem quadraticam perduceres. Ita ad expiscandam tertiam fluxionem ipsius $x y z$, ponatur $m=3$; unde quæsitum incrementum inveniemus hocce,

$$\begin{aligned} x y z + 3 x y z + 3 x y z + 1 x y z \\ + 3 x y z + 6 x y z + 3 x y z \\ + 3 x y z + 3 x y z \\ + 1 x y z \end{aligned}$$

ubi omnes unciaë simul sumptæ sunt eadem, quas reperires trinomium ad altitudinem cubicam evehendo. In genere autem fluxio cujuscunque generis dictæ quantitatis $x y z$ æquabitur sequenti seriei:

$$x y z + \frac{m}{1} \left\{ \begin{matrix} x y \\ x y \end{matrix} \right\} z + \frac{m}{1} \times \frac{m-1}{2} \left\{ \begin{matrix} 1 x y \\ 2 x y \\ 1 x y \end{matrix} \right\} z + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \left\{ \begin{matrix} 1 x y \\ 3 x y \\ 3 x y \\ 1 x y \end{matrix} \right\} z + \&c.$$

E X E M P L U M I I.

Quæritur jam fluens sive incrementum quantitatis trimembris cujuscunque; sive enim hæc quantitatis fluxionalis sit, sive pura, nihil interest. Ita sit investiganda v. g. fluens
 D 2 prima

prima quantitatis $x y z$: In hac specie erit $R=x$, $S=y$, $T=z$,
 $a=b=0$, $c=1$, & $m=-1$; quare juxta primam generalem
 seriem ipsius $x y z$ incrementum primum æquale erit

$$x y z - \begin{Bmatrix} x y \\ 1 \\ x y \end{Bmatrix}^{-1} z + \begin{Bmatrix} 1 x y \\ 1 \\ 2 x y \\ 1 x y \end{Bmatrix}^{-2} z - \begin{Bmatrix} 1 x y \\ 1 \\ 3 x y \\ 3 x y \\ 1 x y \end{Bmatrix}^{-3} z + \begin{Bmatrix} 1 x y \\ 1 \\ 4 x y \\ 6 x y \\ 4 x y \\ 1 x y \end{Bmatrix}^{-4} z - \&c.$$

Et juxta secundam

$$x z y - \begin{Bmatrix} x z \\ 1 \\ x z \end{Bmatrix}^{-1} y + \begin{Bmatrix} 1 x z \\ 1 \\ 2 x z \\ 1 x z \end{Bmatrix}^{-2} y - \begin{Bmatrix} 1 x z \\ 1 \\ 3 x z \\ 3 x z \\ 1 x z \end{Bmatrix}^{-3} y + \begin{Bmatrix} 1 x z \\ 1 \\ 4 x z \\ 6 x z \\ 4 x z \\ 1 x z \end{Bmatrix}^{-4} y - \&c.$$

Vel denique juxta tertiam generalem seriem

$$y z x - \begin{Bmatrix} y z \\ 1 \\ y z \end{Bmatrix}^{-1} x + \begin{Bmatrix} 1 y z \\ 1 \\ 2 y z \\ 1 y z \end{Bmatrix}^{-2} x - \begin{Bmatrix} 1 y z \\ 1 \\ 3 y z \\ 3 y z \\ 1 y z \end{Bmatrix}^{-3} x + \begin{Bmatrix} 1 y z \\ 1 \\ 4 y z \\ 6 y z \\ 4 y z \\ 1 y z \end{Bmatrix}^{-4} x - \&c.$$

Jam, si ordinaria methodo series has investigare cupiamus,
 non parvus adhibendus erit labor: Ad primam itaque seriem
 indagandam ponatur $\boxed{x y z} = x y z + p$, & assumendo fluxio-
 nes

nes erit $\dot{x}y\dot{z} = \dot{x}y\dot{z} + \dot{x}\dot{y}z + \dot{x}y\dot{z} + \dot{p}$, hoc est, $\dot{p} = -\dot{x}\dot{y}z - \dot{x}\dot{y}z$, quare $p = -\boxed{\dot{x}\dot{y}z + \dot{x}\dot{y}z}$, ideoque $\boxed{\dot{x}y\dot{z}} = x\dot{y}z - \boxed{\dot{x}\dot{y}z + \dot{x}\dot{y}z}$. Ponatur jam secundo $\boxed{\dot{x}\dot{y}z + \dot{x}\dot{y}z} = x\dot{y}\dot{z} + \dot{x}y\dot{z} + q$, & capiendo fluxiones erit $\dot{x}\dot{y}\dot{z} + \dot{x}\dot{y}\dot{z} = x\dot{y}\dot{z} + \dot{x}\dot{y}\dot{z} + \dot{x}\dot{y}\dot{z} + \dot{x}\dot{y}\dot{z} + \dot{x}\dot{y}\dot{z} + \dot{q}$, adeoque $\dot{q} = -x\dot{y}\dot{z} - 2x\dot{y}\dot{z} - \dot{x}\dot{y}\dot{z}$; & proinde ipsa quantitas q æquabitur $-\boxed{x\dot{y}\dot{z} + 2x\dot{y}\dot{z} + \dot{x}\dot{y}\dot{z}}$, ac propterea $\boxed{\dot{x}y\dot{z}} = x\dot{y}z - x\dot{y}\dot{z} - \dot{x}y\dot{z} + \boxed{\dot{x}\dot{y}z + 2x\dot{y}\dot{z} + \dot{x}\dot{y}\dot{z}}$. Si cui volupe sit, operationem hac ratione continuare, inveniet, fluentem ipsius $x\dot{y}\dot{z}$ omni ex parte congruere cum prima nostra serie. Quod si quis secundam nostram seriem eodem pacto invenire velit, ponat $\boxed{\dot{x}z\dot{y}} = x\dot{z}\dot{y} + p$, & capiendo fluxiones erit $\dot{x}z\dot{y} = x\dot{z}\dot{y} + \dot{x}z\dot{y} + \dot{x}z\dot{y} + p$, hoc est, $p = -x\dot{z}\dot{y} - \dot{x}z\dot{y}$; & idcirco $p = -\boxed{\dot{x}z\dot{y} + \dot{x}z\dot{y}}$, ac per consequens $\boxed{\dot{x}z\dot{y}} = x\dot{z}\dot{y} - \boxed{\dot{x}z\dot{y} + \dot{x}z\dot{y}}$. Fiat jam secundo $\boxed{\dot{x}z\dot{y} + \dot{x}z\dot{y}} = x\dot{z}\dot{y} + \dot{x}z\dot{y} + q$, & adsumendo fluxiones erit $\dot{x}z\dot{y} + \dot{x}z\dot{y} = x\dot{z}\dot{y} + \dot{x}z\dot{y} + \dot{x}z\dot{y} + \dot{x}z\dot{y} + \dot{x}z\dot{y} + \dot{q}$, adeoque $q = -x\dot{z}\dot{y} - 2x\dot{z}\dot{y} - \dot{x}z\dot{y}$, indeque $q = -\boxed{x\dot{z}\dot{y} + 2x\dot{z}\dot{y} + \dot{x}z\dot{y}}$, unde $\boxed{\dot{x}z\dot{y}} = x\dot{z}\dot{y} - x\dot{z}\dot{y} - \dot{x}z\dot{y} + \boxed{\dot{x}z\dot{y} + 2x\dot{z}\dot{y} + \dot{x}z\dot{y}}$. Operatione hoc modo continuata, secundam nostram seriem invenies. Ad tertiam vero repperiendam ponatur $\boxed{\dot{y}z\dot{x}} = y\dot{z}\dot{x} + p$, & capiendo fluxiones erit $\dot{y}z\dot{x} = y\dot{z}\dot{x} + \dot{y}z\dot{x} + \dot{y}z\dot{x} + p$, id est $p = -y\dot{z}\dot{x} - y\dot{z}\dot{x}$,

$-\dot{y}\dot{z}\dot{x}$, adeoque $p = -\left[\dot{y}\dot{z}\dot{x} + \dot{y}\dot{z}\dot{x}\right]$, ac propterea $\left[\dot{y}\dot{z}\dot{x}\right] =$
 $\dot{y}\dot{z}\dot{x} - \left[\dot{y}\dot{z}\dot{x} + \dot{y}\dot{z}\dot{x}\right]$. Ponatur jam secundo $\left[\dot{y}\dot{z}\dot{x} + \dot{y}\dot{z}\dot{x}\right] = \dot{y}\dot{z}\dot{x}$
 $+ \dot{y}\dot{z}\dot{x} + q$, & adsumendo fluxiones erit $\dot{y}\dot{z}\dot{x} + \dot{y}\dot{z}\dot{x} = \dot{y}\dot{z}\dot{x}$
 $+ \dot{y}\dot{z}\dot{x} + \dot{y}\dot{z}\dot{x} + \dot{y}\dot{z}\dot{x} + \dot{y}\dot{z}\dot{x} + \dot{y}\dot{z}\dot{x} + q$, adeoque $q = -\dot{y}\dot{z}\dot{x}$
 $- 2\dot{y}\dot{z}\dot{x} - \dot{y}\dot{z}\dot{x}$, indeque $q = -\left[\dot{y}\dot{z}\dot{x} + 2\dot{y}\dot{z}\dot{x} + \dot{y}\dot{z}\dot{x}\right]$; unde
 $\left[\dot{y}\dot{z}\dot{x}\right] = \dot{y}\dot{z}\dot{x} - \dot{y}\dot{z}\dot{x} - \dot{y}\dot{z}\dot{x} + \left[\dot{y}\dot{z}\dot{x} + 2\dot{y}\dot{z}\dot{x} + \dot{y}\dot{z}\dot{x}\right]$. Si cui
 operationem hanc profequi lubeat, tertiam nostram seriem in-
 dagabit.

Clariss. Cheynæus quoque in *Fluxionum Methodo inversa* pag.
 52. fluentem ipsius $\dot{z} u y$, seu $\dot{z} x y$ investigare voluit, repre-
 hensus eo nomine a Cl. Moivræo in *Animadvers. ad dict. Tract.*
Cheynæi pag. 72. & seq. quoniam videl. y est quantitas quomo-
 docunque ex indeterminatis & constantibus composita: Verum
 potest eadem ratione, qua fluens ipsius $\dot{y} z$ quæritur, etiam in-
 vestiganda proponi fluens quantitatis $x y \dot{z}$, quam certe ope
 fluentis ipsius $\dot{y} z$ difficulter adinvenires: Et, licet id facile
 Cl. Moivræo largiar, in casu aliquo particulari semper dandam
 esse relationem ipsarum y & z , tamen id nihil impedit, quo-
 minus fluentem ipsius $x y \dot{z}$ investigare liceat; quum, posito
 tantummodo $x = 1$, fluens quantitatis $\dot{y} z$ inveniatur, quemad-
 modum *Exempl. II.* in hac ipsa Propositione videbimus: Inter-
 rim Cl. Cheynæi series tam intricata est, & ex plurium se-
 rierum infinitarum additione constat, ut examinare non lubeat,
 num vel recte eam adinvenerit, vel, quod sæpius ipsi accidisse
 Cl. Moivræus ostendit, ex nimia generalitatis imaginatione a
 vero aberraverit.

E X E M P L U M I I I.

Desideretur incrementum secundum ipsius $x y z$, erit iterum $R=x$, $S=y$, $T=z$, $a=b=0$, $c=1$, at $m=-2$; unde juxta primam seriem invenies:

$$x y z - 2 \begin{Bmatrix} x y \\ x y \\ x y \end{Bmatrix} z + 3 \begin{Bmatrix} 1 x y \\ 2 x y \\ 1 x y \end{Bmatrix} z - 4 \begin{Bmatrix} 1 x y \\ 3 x y \\ 3 x y \\ 1 x y \end{Bmatrix} z + 5 \begin{Bmatrix} 1 x y \\ 4 x y \\ 6 x y \\ 4 x y \\ 1 x y \end{Bmatrix} z - \&c.$$

Et juxta secundam:

$$x z y - 2 \begin{Bmatrix} x z \\ x z \\ x z \end{Bmatrix} y + 3 \begin{Bmatrix} 1 x z \\ 2 x z \\ 1 x z \end{Bmatrix} y - 4 \begin{Bmatrix} 1 x z \\ 3 x z \\ 3 x z \\ 1 x z \end{Bmatrix} y + 5 \begin{Bmatrix} 1 x z \\ 4 x z \\ 6 x z \\ 4 x z \\ 1 x z \end{Bmatrix} y - \&c.$$

Ac denique juxta tertiam:

$$y z x - 2 \begin{Bmatrix} y z \\ y z \\ y z \end{Bmatrix} x + 3 \begin{Bmatrix} 1 y z \\ 2 y z \\ 1 y z \end{Bmatrix} x - 4 \begin{Bmatrix} 1 y z \\ 3 y z \\ 3 y z \\ 1 y z \end{Bmatrix} x + 5 \begin{Bmatrix} 1 y z \\ 4 y z \\ 6 y z \\ 4 y z \\ 1 y z \end{Bmatrix} x - \&c.$$

Si hoc ipsum vulgata methodo præstare velimus, recurrendum erit ad fluentem primam ipsius $x y z$, quam superiori exemplo adinvenimus, & deinde singulorum terminorum aut etiam singularum columnarum rursus fluentes capiendæ, hæque simul addendæ sequentem in modum.

$$\begin{aligned}
 \boxed{xyz} &= xyz - \left\{ \begin{matrix} xy \\ 1 \end{matrix} \right\} z + \left\{ \begin{matrix} 1 & xy \\ 2 & xy \\ 1 & xy \end{matrix} \right\} z - \left\{ \begin{matrix} 1 & xy \\ 3 & xy \\ 3 & xy \\ 1 & xy \end{matrix} \right\} z + \&c. \\
 - \boxed{\begin{matrix} 1 & 1 \\ xy & z \\ 1 & 1 \end{matrix}} &= - \left\{ \begin{matrix} xy \\ 1 \end{matrix} \right\} z + \left\{ \begin{matrix} 1 & xy \\ 2 & xy \\ 1 & xy \end{matrix} \right\} z - \left\{ \begin{matrix} 1 & xy \\ 3 & xy \\ 3 & xy \\ 1 & xy \end{matrix} \right\} z + \&c. \\
 + \boxed{\begin{matrix} 2 & 2 \\ 1 & xy & z \\ 1 & 1 & 2 \\ 2 & 2 \\ 1 & xy & z \end{matrix}} &= + \left\{ \begin{matrix} 1 & xy \\ 2 & xy \\ 1 & xy \end{matrix} \right\} z - \left\{ \begin{matrix} 1 & xy \\ 3 & xy \\ 3 & xy \\ 1 & xy \end{matrix} \right\} z + \&c. \\
 - \boxed{\begin{matrix} 3 & 3 \\ 1 & xy & z \\ 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 3 \\ 1 & xy & z \end{matrix}} &= - \left\{ \begin{matrix} 1 & xy \\ 3 & xy \\ 3 & xy \\ 1 & xy \end{matrix} \right\} z + \&c. \\
 + \&c. & \qquad \qquad \qquad + \&c. \quad (\text{add.})
 \end{aligned}$$

$$\boxed{\boxed{xyz}} = xyz - 2 \left\{ \begin{matrix} xy \\ 1 \end{matrix} \right\} z + 3 \left\{ \begin{matrix} 1 & xy \\ 2 & xy \\ 1 & xy \end{matrix} \right\} z - 4 \left\{ \begin{matrix} 1 & xy \\ 3 & xy \\ 3 & xy \\ 1 & xy \end{matrix} \right\} z + \&c.$$

Id quod omnino convenit cum prima nostra serie ex universali deducta. Quod si secundam eadem methodo adinvenire velimus, iterum recurrendum erit ad fluentem primi generis, quam convenien-

nienter secundæ nostræ seriei in anteriori Exemplo invenimus, & singularum columnarum rursus fluentes capiendæ & invicem addendæ hac ratione:

$$\begin{array}{l}
 \boxed{\begin{array}{c} \overset{1-1}{xzy} \end{array}} = xzy - \left\{ \begin{array}{c} \overset{2}{xz} \\ \overset{1}{1} \end{array} \right\}^{-3} y + \left\{ \begin{array}{c} \overset{3}{1xz} \\ \overset{1}{1} \end{array} \right\}^{-4} y - \left\{ \begin{array}{c} \overset{4}{1xz} \\ \overset{1}{1} \end{array} \right\}^{-5} y + \&c. \\
 \\
 - \boxed{\begin{array}{c} \overset{2-2}{xzy} \\ \overset{1}{1} \end{array}} = - \left\{ \begin{array}{c} \overset{2}{xz} \\ \overset{1}{1} \end{array} \right\}^{-3} y + \left\{ \begin{array}{c} \overset{3}{1xz} \\ \overset{1}{1} \end{array} \right\}^{-4} y - \left\{ \begin{array}{c} \overset{4}{1xz} \\ \overset{1}{1} \end{array} \right\}^{-5} y + \&c. \\
 \\
 + \boxed{\begin{array}{c} \overset{3-3}{1xzy} \\ \overset{1}{1} \end{array}} = + \left\{ \begin{array}{c} \overset{3}{1xz} \\ \overset{1}{1} \end{array} \right\}^{-4} y - \left\{ \begin{array}{c} \overset{4}{1xz} \\ \overset{1}{1} \end{array} \right\}^{-5} y + \&c. \\
 \\
 - \boxed{\begin{array}{c} \overset{4-4}{1xzy} \\ \overset{1}{1} \end{array}} = - \left\{ \begin{array}{c} \overset{4}{1xz} \\ \overset{1}{1} \end{array} \right\}^{-5} y + \&c. \\
 \\
 + \&c. \qquad \qquad \qquad + \&c. \qquad \qquad \qquad \text{(add.)}
 \end{array}$$

$$\boxed{\boxed{\overset{1}{xzy}}} = xzy - 2 \left\{ \begin{array}{c} \overset{2}{xz} \\ \overset{1}{1} \end{array} \right\}^{-3} y + 3 \left\{ \begin{array}{c} \overset{3}{1xz} \\ \overset{1}{1} \end{array} \right\}^{-4} y - 4 \left\{ \begin{array}{c} \overset{4}{1xz} \\ \overset{1}{1} \end{array} \right\}^{-5} y + \&c.$$

Si denique tertiam seriem eodem modo indagare velimus, iterum

rum adsumenda est fluens prima ipsius yzx congruenter tertiæ seriei in priori Exemplo inventa, ac similiter singularum columnarum fluentes capiendæ, & addendæ hoc pacto:

$$\begin{array}{rcl}
 \boxed{\begin{array}{c} 1 \quad 1 \\ yz x \end{array}} & = yz x - \left\{ \begin{array}{c} 2 \\ yz \\ 1 \quad 1 \\ yz \end{array} \right\} x + \left\{ \begin{array}{c} 3 \\ 1yz \\ 1 \quad 2 \\ 2yz \\ 2 \quad 1 \\ 1yz \end{array} \right\} x - \left\{ \begin{array}{c} 4 \\ 1yz \\ 1 \quad 3 \\ 3yz \\ 2 \quad 2 \\ 3yz \\ 3 \quad 1 \\ 1yz \end{array} \right\} x + \&c. \\
 - \boxed{\begin{array}{c} 2 \quad 2 \\ yz x \\ 1 \quad 1 \quad 2 \\ yz x \end{array}} & = - \left\{ \begin{array}{c} 2 \\ yz \\ 1 \quad 1 \\ yz \end{array} \right\} x + \left\{ \begin{array}{c} 3 \\ 1yz \\ 1 \quad 2 \\ 2yz \\ 2 \quad 1 \\ 1yz \end{array} \right\} x - \left\{ \begin{array}{c} 4 \\ 1yz \\ 1 \quad 3 \\ 3yz \\ 2 \quad 2 \\ 3yz \\ 3 \quad 1 \\ 1yz \end{array} \right\} x + \&c. \\
 + \boxed{\begin{array}{c} 3 \quad 3 \\ 1yz x \\ 1 \quad 2 \quad 3 \\ + 2yz x \\ 2 \quad 1 \quad 3 \\ + 1yz x \end{array}} & = + \left\{ \begin{array}{c} 3 \\ 1yz \\ 1 \quad 2 \\ 2yz \\ 2 \quad 1 \\ 1yz \end{array} \right\} x - \left\{ \begin{array}{c} 4 \\ 1yz \\ 1 \quad 3 \\ 3yz \\ 2 \quad 2 \\ 3yz \\ 3 \quad 1 \\ 1yz \end{array} \right\} x + \&c. \\
 - \boxed{\begin{array}{c} 4 \quad 4 \\ 1yz x \\ 1 \quad 3 \quad 4 \\ + 3yz x \\ 2 \quad 2 \quad 4 \\ + 3yz x \\ 3 \quad 1 \quad 4 \\ + 1yz x \end{array}} & = - \left\{ \begin{array}{c} 4 \\ 1yz \\ 1 \quad 3 \\ 3yz \\ 2 \quad 2 \\ 3yz \\ 3 \quad 1 \\ 1yz \end{array} \right\} x + \&c. \\
 + \&c. & & + \&c.
 \end{array}$$

(add.)

$$\boxed{\begin{array}{c} 1 \\ yz x \end{array}} = yz x - 2 \left\{ \begin{array}{c} 2 \\ yz \\ 1 \quad 1 \\ yz \end{array} \right\} x + 3 \left\{ \begin{array}{c} 3 \\ 1yz \\ 1 \quad 2 \\ 2yz \\ 2 \quad 1 \\ 1yz \end{array} \right\} x - 4 \left\{ \begin{array}{c} 4 \\ 1yz \\ 1 \quad 3 \\ 3yz \\ 2 \quad 2 \\ 3yz \\ 3 \quad 1 \\ 1yz \end{array} \right\} x + \&c.$$

EXEM-

E X E M P L U M I V.

Investiganda sit fluens tertia ipsius $x y z$, erit $a = b = 0$, $c = 1$, & $m = -3$; unde juxta primam seriem fluens quaesita aequabitur

$$x y z - 3 \begin{Bmatrix} x y \\ x y \\ x y \end{Bmatrix} z + 6 \begin{Bmatrix} 1 x y \\ 2 x y \\ 1 x y \end{Bmatrix} z - 10 \begin{Bmatrix} 1 x y \\ 3 x y \\ 3 x y \\ 1 x y \end{Bmatrix} z + \&c.$$

Ac juxta secundam

$$x z y - 3 \begin{Bmatrix} x z \\ x z \\ x z \end{Bmatrix} y + 6 \begin{Bmatrix} 1 x z \\ 2 x z \\ 1 x z \end{Bmatrix} y - 10 \begin{Bmatrix} 1 x z \\ 3 x z \\ 3 x z \\ 1 x z \end{Bmatrix} y + \&c.$$

Et juxta tertiam

$$y z x - 3 \begin{Bmatrix} y z \\ y z \\ y z \end{Bmatrix} x + 6 \begin{Bmatrix} 1 y z \\ 2 y z \\ 1 y z \end{Bmatrix} x - 10 \begin{Bmatrix} 1 y z \\ 3 y z \\ 3 y z \\ 1 y z \end{Bmatrix} x + \&c.$$

In genere vero fluens quaelibet quantitatis $x y z$ æqualis est sequenti seriei:

$$x y z + \frac{m}{1} \begin{Bmatrix} x y \\ x y \\ x y \end{Bmatrix} z + \frac{m}{1} \times \frac{m-1}{2} \begin{Bmatrix} 1 x y \\ 2 x y \\ 1 x y \end{Bmatrix} z + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \begin{Bmatrix} 1 x y \\ 3 x y \\ 3 x y \\ 1 x y \end{Bmatrix} z + \&c.$$

Aut etiam huic:

$$x^{\frac{1}{1}} z^{\frac{1}{1}} y^{\frac{1}{1}} + \frac{m}{1} \left\{ \begin{matrix} x^{\frac{2}{1}} z^{\frac{2}{1}} \\ \frac{1}{1} x^{\frac{1}{1}} z^{\frac{1}{1}} \\ x^{\frac{2}{1}} z^{\frac{1}{1}} \end{matrix} \right\}^{\frac{m-1}{1}} y^{\frac{1}{1}} + \frac{m}{1} \times \frac{m-1}{2} \left\{ \begin{matrix} 1 x^{\frac{3}{1}} z^{\frac{3}{1}} \\ \frac{1}{1} x^{\frac{2}{1}} z^{\frac{2}{1}} \\ 2 x^{\frac{2}{1}} z^{\frac{1}{1}} \\ \frac{2}{1} x^{\frac{1}{1}} z^{\frac{1}{1}} \\ 1 x^{\frac{3}{1}} z^{\frac{1}{1}} \end{matrix} \right\}^{\frac{m-2}{1}} y^{\frac{1}{1}} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \left\{ \begin{matrix} 1 x^{\frac{4}{1}} z^{\frac{4}{1}} \\ \frac{1}{1} x^{\frac{3}{1}} z^{\frac{3}{1}} \\ 3 x^{\frac{2}{1}} z^{\frac{2}{1}} \\ \frac{2}{1} x^{\frac{2}{1}} z^{\frac{1}{1}} \\ 3 x^{\frac{2}{1}} z^{\frac{1}{1}} \\ \frac{3}{1} x^{\frac{1}{1}} z^{\frac{1}{1}} \\ 1 x^{\frac{4}{1}} z^{\frac{1}{1}} \end{matrix} \right\}^{\frac{m-3}{1}} y^{\frac{1}{1}} + \&c.$$

Vel denique huic:

$$y^{\frac{1}{1}} z^{\frac{1}{1}} x^{\frac{1}{1}} + \frac{m}{1} \left\{ \begin{matrix} y^{\frac{2}{1}} z^{\frac{2}{1}} \\ \frac{1}{1} y^{\frac{1}{1}} z^{\frac{1}{1}} \\ y^{\frac{2}{1}} z^{\frac{1}{1}} \end{matrix} \right\}^{\frac{m-1}{1}} x^{\frac{1}{1}} + \frac{m}{1} \times \frac{m-1}{2} \left\{ \begin{matrix} 1 y^{\frac{3}{1}} z^{\frac{3}{1}} \\ \frac{1}{1} y^{\frac{2}{1}} z^{\frac{2}{1}} \\ 2 y^{\frac{2}{1}} z^{\frac{1}{1}} \\ \frac{2}{1} y^{\frac{1}{1}} z^{\frac{1}{1}} \\ 1 y^{\frac{3}{1}} z^{\frac{1}{1}} \end{matrix} \right\}^{\frac{m-2}{1}} x^{\frac{1}{1}} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \left\{ \begin{matrix} 1 y^{\frac{4}{1}} z^{\frac{4}{1}} \\ \frac{1}{1} y^{\frac{3}{1}} z^{\frac{3}{1}} \\ 3 y^{\frac{2}{1}} z^{\frac{2}{1}} \\ \frac{2}{1} y^{\frac{2}{1}} z^{\frac{1}{1}} \\ 3 y^{\frac{2}{1}} z^{\frac{1}{1}} \\ \frac{3}{1} y^{\frac{1}{1}} z^{\frac{1}{1}} \\ 1 y^{\frac{4}{1}} z^{\frac{1}{1}} \end{matrix} \right\}^{\frac{m-3}{1}} x^{\frac{1}{1}} + \&c.$$

E X E M P L U M V.

Per has porro series etiam data fluxione vel fluente aliqua ipsa quantitatis inveniri potest: Ut si detur

$$\begin{aligned} & x^{\frac{3}{1}} y^{\frac{1}{1}} z^{\frac{1}{1}} + 3 x^{\frac{2}{1}} y^{\frac{2}{1}} z^{\frac{1}{1}} + 3 x^{\frac{2}{1}} y^{\frac{1}{1}} z^{\frac{2}{1}} + 1 x^{\frac{3}{1}} y^{\frac{1}{1}} z^{\frac{1}{1}} \\ & + 3 x^{\frac{2}{1}} y^{\frac{1}{1}} z^{\frac{2}{1}} + 6 x^{\frac{1}{1}} y^{\frac{2}{1}} z^{\frac{2}{1}} + 3 x^{\frac{1}{1}} y^{\frac{1}{1}} z^{\frac{3}{1}} \\ & + 3 x^{\frac{2}{1}} y^{\frac{1}{1}} z^{\frac{1}{1}} + 3 x^{\frac{1}{1}} y^{\frac{2}{1}} z^{\frac{1}{1}} \\ & + 1 x^{\frac{3}{1}} y^{\frac{1}{1}} z^{\frac{1}{1}} \end{aligned}$$

ejusque fluxionis desideretur reversio: Apparet satis ex coëfficientibus, esse hanc fluxionem tertiam quantitatis trimembris; quare capiendo tantum singulorum terminorum fluentem tertiam invenietur ipsa quantitas quæsitæ xyz . Verum longe fa-

cilius id ipsum præstari poterit, si ipsius $\frac{m}{a \cdot b \cdot c} xyz$ (quam sub initium hujus Propositionis in infinita serie tribus modis exhibuimus)

mus) rursus fluxionem vel fluentem generis p capiamus, sicque

$\frac{m+p}{a \cdot b \cdot c} x y z$ æquabitur sequenti seriei:

$$\frac{a \cdot b \cdot c}{x y z} + \frac{m+p}{1} \left\{ \begin{array}{c} a \cdot b+1 \\ x y \\ a+1 \cdot b \\ x y \end{array} \right\} z^{c+m+p-1} + \frac{m+p}{1} \times \frac{m+p-1}{2} \left\{ \begin{array}{c} a \cdot b+2 \\ 1 x y \\ a+1 \cdot b+1 \\ 2 x y \\ a+2 \cdot b \\ 1 x y \end{array} \right\} z^{c+m+p-2} + \&c.$$

Vel etiam huic:

$$\frac{a \cdot c \cdot b}{x z y} + \frac{m+p}{1} \left\{ \begin{array}{c} a \cdot c+1 \\ x z \\ a+1 \cdot c \\ x z \end{array} \right\} y^{b+m+p-1} + \frac{m+p}{1} \times \frac{m+p-1}{2} \left\{ \begin{array}{c} a \cdot c+2 \\ 1 x z \\ a+1 \cdot c+1 \\ 2 x z \\ a+2 \cdot c \\ 1 x z \end{array} \right\} y^{b+m+p-2} + \&c.$$

Aud denique huic:

$$\frac{b \cdot c \cdot a}{y z x} + \frac{m+p}{1} \left\{ \begin{array}{c} b \cdot c+1 \\ y z \\ b+1 \cdot c \\ y z \end{array} \right\} x^{a+m+p-1} + \frac{m+p}{1} \times \frac{m+p-1}{2} \left\{ \begin{array}{c} b \cdot c+2 \\ 1 y z \\ b+1 \cdot c+1 \\ 2 y z \\ b+2 \cdot c \\ 1 y z \end{array} \right\} x^{a+m+p-2} + \&c.$$

Quare in hoc casu, quoniam m ab initio æqualis erat 3, jam vero adsumendo $p = -3$, erit $m+p=0$, ideoque habebimus

$\frac{a \cdot b \cdot c}{x y z}$, sive (quoniam ex data fluxione comparata cum generali serie apparet, hoc loco $a=b=c=0$ esse) $x y z$.

EXEMPLUM VI.

Similiter, si fluens cujuscunque altitudinis detur, poterit per eam ipsa quantitas inveniri: Ut si detur

E 3

$\frac{-1}{x y z}$

$$x y z^{-1} - 2 \begin{Bmatrix} x y \\ 1 \\ x y \end{Bmatrix} z^{-2} + 3 \begin{Bmatrix} 1 x y \\ 2 x y \\ 1 x y \end{Bmatrix} z^{-3} - 4 \begin{Bmatrix} 1 x y \\ 3 x y \\ 3 x y \\ 1 x y \end{Bmatrix} z^{-4} + \&c.$$

vel etiam

$$x z y^{-1} - 2 \begin{Bmatrix} x z \\ 1 \\ x z \end{Bmatrix} y^{-2} + 3 \begin{Bmatrix} 1 x z \\ 2 x z \\ 1 x z \end{Bmatrix} y^{-3} - 4 \begin{Bmatrix} 1 x z \\ 3 x z \\ 3 x z \\ 1 x z \end{Bmatrix} y^{-4} + \&c.$$

aut

$$y z x^{-1} - 2 \begin{Bmatrix} y z \\ 1 \\ y z \end{Bmatrix} x^{-2} + 3 \begin{Bmatrix} 1 y z \\ 2 y z \\ 1 y z \end{Bmatrix} x^{-3} - 4 \begin{Bmatrix} 1 y z \\ 3 y z \\ 3 y z \\ 1 y z \end{Bmatrix} x^{-4} + \&c.$$

ejusque fluentis reversio sit investiganda: Apparet ex coëfficientibus, esse hanc fluentem secundam quantitatis trimembris, unde $m = -2$, & comparando hasce series cum generalioribus sub initium hujus Propositionis exhibitis patet, $a = 0$, $b = 0$, $c = 1$ esse; jam ad obtinendam ipsam quantitatem poni debet $p = 2$, ita ut $m + p$ nihilo sit æquale; unde, quum reliqui termini evanescant, restabit duntaxat $x y z$, id est in hoc casu, $x y z$.

EXEM.

E X E M P L U M V I I.

EAdem ratione possunt tum fluxionum tum fluentium decre-
 menta & incrementa quævis inveniri: Sic e. g. si datæ
 fluxionis quartæ rursus fluxio sexta capienda esset, foret $m=4$,
 & poni deberet $p=6$, adeoque $m+p=10$, & comparando
 duntaxat datam fluxionem cum generali serie innotesceret va-
 lor indicum fluxionalium, adeoque ope seriei *Exempl. 5.* pro-
 positæ statim desiderata fluxio reperiri posset. Similiter, si da-
 tæ fluxionis tertię fluens secunda investiganda esset, foret $m=3$,
 & ponendo $p=-2$, esset $m+p=1$, sicque fluxio prima ipsius
 reversionis indagaretur; at si fluens tertia desideraretur, ubi
 proinde $p=-3$, jam foret $m+p=0$, sicque in ipsam quantita-
 tem incideres, congruenter *Exempl. 5.* quod si vero fluens e.
 g. quinta inveniendâ proponeretur, ubi $p=-5$, jam $m+p$ æ-
 quale esset -2 , sicque secundi generis fluentem ipsius quantita-
 tis invenires.

E X E M P L U M V I I I.

Similiter, si fluentis alicujus, pone tertię, iterum fluens quæ-
 dam e. g. secunda adsumenda esset, foret $m=-3$, $p=-2$,
 proinde $m+p=-5$, sicque ipsius quantitatis fluens quinti ordi-
 nis innotesceret. Ita etiam, si datæ fluentis quartæ fluxio sep-
 ma reperienda esset, foret $m=-4$, $p=7$, unde $m+p=3$,
 ideoque in tertiam fluxionem ipsius quantitatis incideres; quod
 si fluxio quarta desideraretur, ubi $p=4$, & consequenter
 $m+p=0$, jam ipsam quantitatem invenires, perinde ac in
Exempl. 6. at si fluxio secunda investiganda proponeretur,
 ubi $p=2$, adeoque $m+p=-2$, jam secundam fluentem ipsius
 quantitatis repperires.

EXEM-

E X E M P L U M I X.

SI quis invenire desideret incrementum vel decrementum quantitatis alicujus trium dimensionum, puta $\dot{x}^{\frac{a}{3}}$, hoc est $\dot{x} \dot{x} \dot{x}$, tum erit $R=S=T=x$, & $a=b=c$; ideoque erit fluens vel fluxio qualiscunque

$$\begin{aligned} & \dot{x}^{\frac{a}{2}} \dot{x}^{a+m} + \frac{m}{1} \times 2 \dot{x}^{\frac{a}{2}} \dot{x}^{a+1} \dot{x}^{a+m-1} + \frac{m}{1} \times \frac{m-1}{2} \left\{ \begin{array}{cc} \dot{x}^{\frac{a}{2}} \dot{x}^{a+2} \\ 2 \dot{x}^{\frac{a+1}{2}} \dot{x}^{a+1} \end{array} \right\} \dot{x}^{a+m-2} \\ & + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \left\{ \begin{array}{cc} \dot{x}^{\frac{a}{2}} \dot{x}^{a+3} \\ 2 \dot{x}^{\frac{a+1}{2}} \dot{x}^{a+2} \\ 6 \dot{x}^{\frac{a+1}{2}} \dot{x}^2 \end{array} \right\} \dot{x}^{a+m-3} + \&c. \end{aligned}$$

& sic una tantum prodit series, tam quoad fluentes, quam fluxiones.

E X E M P L U M X.

Q Uod si ope seriei nostræ investiganda sit fluens prima ipsius $\dot{x}^3$, erit $R=S=T=x$, $a=b=0$, $c=1$, & $m=-1$, & proinde juxta primam seriem fluens quæsitæ æquabitur

$$3\dot{x}\dot{x}\dot{x} - 6\dot{x}\dot{x}\dot{x} + \left\{ \begin{array}{cc} 6\dot{x}\dot{x}^2 \\ 6\dot{x}\dot{x} \end{array} \right\} \dot{x} - \left\{ \begin{array}{cc} 6\dot{x}\dot{x}^3 \\ 18\dot{x}\dot{x} \end{array} \right\} \dot{x} + \&c.$$

& juxta secundam ac tertiam:

$$3\dot{x}\dot{x}\dot{x} - \left\{ \begin{array}{cc} 3\dot{x}\dot{x}^2 \\ 3\dot{x}\dot{x} \end{array} \right\} \dot{x} + \left\{ \begin{array}{cc} 3\dot{x}\dot{x}^3 \\ 9\dot{x}\dot{x} \end{array} \right\} \dot{x} - \&c.$$

Et

Et quoniam hæ duæ series inter se æquales sunt, erit etiam

$$3xxx = 9xxx - \left\{ \begin{matrix} 9xx \\ 9xx \end{matrix} \right\} x + \left\{ \begin{matrix} 9xx \\ 27xx \end{matrix} \right\} x - \&c.$$

ac dividendo per 3, erit

$$xxx = 3xxx - \left\{ \begin{matrix} 3xx \\ 3xx \end{matrix} \right\} x + \left\{ \begin{matrix} 3xx \\ 9xx \end{matrix} \right\} x - \&c.$$

quam seriem infinitam si vocemus Q , habebimus juxta primam seriem ipsius $3xxx$ fluentem primam æqualem $3xxx - 2Q$, hoc est (quoniam jam Q æquatur ipfi xxx), $3xxx - 2xxx$, five xxx , & juxta secundam & tertiam seriem statim habebimus Q , adeoque xxx , quod proinde est incrementum primum ipsius $3xxx$.

E X E M P L U M X I.

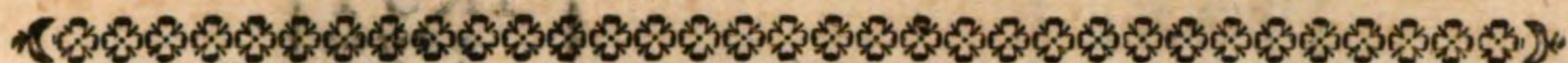
SI quis in universali nostra serie pro quantitate R , S , vel T , (quæ omnes ibi ut variabiles consideratæ fuerunt) adhibere velit quantitatem constantem, pone n , tum duæ tantummodo series prodibunt, eaque, quæ incrementa vel decrements ipsius n cæteroquin exhiberet, locum jam non habebit: Ita ad inveniendam sive fluxionem sive fluentem qualemcunque ipsius nxy , erit $R=x$, $S=y$, $T=n$, $c=0$; quare juxta secundam seriem (nam prima ob rationem modo dictam in hoc

casu locum non invenit) æquabitur $nxy + \frac{m}{1} \times nxy + \frac{m}{1} \times \frac{m-1}{2} \times nxy + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times nxy + \&c.$ reliqui enim termini, qui in quavis columna reperiuntur, jam evanescunt, quoniam quantitas n neque fluxiones neque fluentes admittit: Juxta tertiam

tiam vero feriem habebimus $n \overset{b \ a+m}{y} \overset{b+1 \ a+m-1}{x} + \frac{m}{1} \times n \overset{b+1 \ a+m-1}{y} \overset{b+2 \ a+m-2}{x} + \frac{m}{1} \times \frac{m-1}{2} \times$
 $n \overset{b+2 \ a+m-2}{y} \overset{b+3 \ a+m-3}{x} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times n \overset{b+3 \ a+m-3}{y} \overset{b+4 \ a+m-4}{x} + \&c.$ nam & hic reliqui in quali-
 bet columna termini eadem de causa evanescunt. Posito jam

$n = 1$, erit $\overset{\frac{m}{a \ b}}{x} \overset{a \ b+m}{y} = \overset{\frac{m}{a \ b}}{x} \overset{a \ b+m}{y} + \frac{m}{1} \times \overset{a+1 \ b+m-1}{x} \overset{a+2 \ b+m-2}{y} + \frac{m}{1} \times \frac{m-1}{2} \times \overset{a+2 \ b+m-2}{x} \overset{a+3 \ b+m-3}{y} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \overset{a+3 \ b+m-3}{x} \overset{a+4 \ b+m-4}{y} + \&c.$ vel $\overset{b \ a+m}{y} \overset{b+1 \ a+m-1}{x} + \frac{m}{1} \times \overset{b+1 \ a+m-1}{y} \overset{b+2 \ a+m-2}{x} + \frac{m}{1} \times \frac{m-1}{2} \times \overset{b+2 \ a+m-2}{y} \overset{b+3 \ a+m-3}{x} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \overset{b+3 \ a+m-3}{y} \overset{b+4 \ a+m-4}{x} + \&c.$ & sic rursus eadem prodeunt series, quas sub ini-
 tium anterioris Propositionis exhibuimus.





PROPOSITIO III.

*Quatuor quantitatum variabilium in semet multiplicatarum,
tam fluxiones quam fluentes cujuscunque ordinis
in serie infinita exhibere.*

SIt $\overset{a}{R} \overset{b}{S} \overset{c}{T} \overset{d}{V}$ productum ex quatuor quantitibus fluxiona-
libus constans, ejusque sit varietas (hoc est, qua decre-
mentum, qua incrementum) generis (m) indaganda, dico,
ipsum $\overset{a}{R} \overset{b}{S} \overset{c}{T} \overset{d}{V}$ æquale fore sequenti seriei infinitæ:

$$\overset{a}{R} \overset{b}{S} \overset{c}{T} \overset{d+m}{V} + \frac{m}{1} \left\{ \begin{matrix} \overset{a}{R} \overset{b}{S} \overset{c+1}{T} \\ \overset{a}{R} \overset{b+1}{S} \overset{c}{T} \\ \overset{a+1}{R} \overset{b}{S} \overset{c}{T} \end{matrix} \right\} \overset{d+m-1}{V} + \frac{m}{1} \times \frac{m-1}{2} \left\{ \begin{matrix} \overset{a}{R} \overset{b}{S} \overset{c+2}{T} \\ \overset{a}{R} \overset{b+1}{S} \overset{c+1}{T} \\ \overset{a+1}{R} \overset{b}{S} \overset{c+1}{T} \\ \overset{a}{R} \overset{b+2}{S} \overset{c}{T} \\ \overset{a+1}{R} \overset{b+1}{S} \overset{c}{T} \\ \overset{a+2}{R} \overset{b}{S} \overset{c}{T} \end{matrix} \right\} \overset{d+m-2}{V}$$

$$\begin{aligned}
 & + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \left\{ \begin{array}{l} \overset{a}{1} \overset{b}{R} \overset{c+3}{S} \overset{d}{T} \\ \overset{a}{3} \overset{b+1}{R} \overset{c+2}{S} \overset{d}{T} \\ \overset{a+1}{3} \overset{b}{R} \overset{c+2}{S} \overset{d}{T} \\ \overset{a}{3} \overset{b+2}{R} \overset{c+1}{S} \overset{d}{T} \\ \overset{a+1}{6} \overset{b+1}{R} \overset{c+1}{S} \overset{d}{T} \\ \overset{a+2}{3} \overset{b}{R} \overset{c+1}{S} \overset{d}{T} \\ \overset{a}{1} \overset{b+3}{R} \overset{c}{S} \overset{d}{T} \\ \overset{a+1}{3} \overset{b+2}{R} \overset{c}{S} \overset{d}{T} \\ \overset{a+2}{3} \overset{b+1}{R} \overset{c}{S} \overset{d}{T} \\ \overset{a+3}{1} \overset{b}{R} \overset{c}{S} \overset{d}{T} \end{array} \right\} \overset{d+m-3}{V} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \left\{ \begin{array}{l} \overset{a}{1} \overset{b}{R} \overset{c+4}{S} \overset{d}{T} \\ \overset{a}{4} \overset{b+1}{R} \overset{c+3}{S} \overset{d}{T} \\ \overset{a+1}{4} \overset{b}{R} \overset{c+3}{S} \overset{d}{T} \\ \overset{a}{6} \overset{b+2}{R} \overset{c+2}{S} \overset{d}{T} \\ \overset{a+1}{12} \overset{b+1}{R} \overset{c+2}{S} \overset{d}{T} \\ \overset{a+2}{6} \overset{b}{R} \overset{c+2}{S} \overset{d}{T} \\ \overset{a}{4} \overset{b+3}{R} \overset{c+1}{S} \overset{d}{T} \\ \overset{a+1}{12} \overset{b+2}{R} \overset{c+1}{S} \overset{d}{T} \\ \overset{a+2}{12} \overset{b+1}{R} \overset{c+1}{S} \overset{d}{T} \\ \overset{a+3}{4} \overset{b}{R} \overset{c+1}{S} \overset{d}{T} \\ \overset{a}{1} \overset{b+4}{R} \overset{c}{S} \overset{d}{T} \\ \overset{a+1}{4} \overset{b+3}{R} \overset{c}{S} \overset{d}{T} \\ \overset{a+2}{6} \overset{b+2}{R} \overset{c}{S} \overset{d}{T} \\ \overset{a+3}{4} \overset{b+1}{R} \overset{c}{S} \overset{d}{T} \\ \overset{a+4}{1} \overset{b}{R} \overset{c}{S} \overset{d}{T} \end{array} \right\} \overset{d+m-4}{V} + \&c.
 \end{aligned}$$

Vel etiam huic:

$$\begin{aligned}
 & \overset{a}{R} \overset{b}{S} \overset{d}{V} \overset{c+m}{T} + \frac{m}{1} \left\{ \begin{array}{l} \overset{a}{R} \overset{b}{S} \overset{d+1}{V} \\ \overset{a}{R} \overset{b+1}{S} \overset{d}{V} \\ \overset{a+1}{R} \overset{b}{S} \overset{d}{V} \end{array} \right\} \overset{c+m-1}{T} + \frac{m}{1} \times \frac{m-1}{2} \left\{ \begin{array}{l} \overset{a}{1} \overset{b}{R} \overset{d+2}{S} \overset{V}{V} \\ \overset{a}{2} \overset{b+1}{R} \overset{d+1}{S} \overset{V}{V} \\ \overset{a+1}{2} \overset{b}{R} \overset{d+1}{S} \overset{V}{V} \\ \overset{a}{1} \overset{b+2}{R} \overset{d}{S} \overset{V}{V} \\ \overset{a+1}{2} \overset{b+1}{R} \overset{d}{S} \overset{V}{V} \\ \overset{a+2}{1} \overset{b}{R} \overset{d}{S} \overset{V}{V} \end{array} \right\} \overset{c+m-2}{T} \\
 & + \frac{m}{1} \times
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \left\{ \begin{array}{l} 1 \dot{R} \dot{S} \dot{V} \\ a \quad b+1 \quad d+2 \\ 3 \dot{R} \dot{S} \dot{V} \\ a+1 \quad b \quad d+2 \\ 3 \dot{R} \dot{S} \dot{V} \\ a \quad b+2 \quad d+1 \\ 3 \dot{R} \dot{S} \dot{V} \\ a+1 \quad b+1 \quad d+1 \\ 6 \dot{R} \dot{S} \dot{V} \\ a+2 \quad b \quad d+1 \\ 3 \dot{R} \dot{S} \dot{V} \\ a \quad b+3 \quad d \\ 1 \dot{R} \dot{S} \dot{V} \\ a+1 \quad b+2 \quad d \\ 3 \dot{R} \dot{S} \dot{V} \\ a+2 \quad b+1 \quad d \\ 3 \dot{R} \dot{S} \dot{V} \\ a+3 \quad b \quad d \\ 1 \dot{R} \dot{S} \dot{V} \end{array} \right\} T^{c+m-3} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \left\{ \begin{array}{l} 1 \dot{R} \dot{S} \dot{V} \\ a \quad b \quad d+4 \\ 4 \dot{R} \dot{S} \dot{V} \\ a \quad b+1 \quad d+3 \\ 4 \dot{R} \dot{S} \dot{V} \\ a+1 \quad b \quad d+3 \\ 6 \dot{R} \dot{S} \dot{V} \\ a \quad b+2 \quad d+2 \\ 6 \dot{R} \dot{S} \dot{V} \\ a+1 \quad b+1 \quad d+2 \\ 12 \dot{R} \dot{S} \dot{V} \\ a+2 \quad b \quad d+2 \\ 6 \dot{R} \dot{S} \dot{V} \\ a \quad b+3 \quad d+1 \\ 4 \dot{R} \dot{S} \dot{V} \\ a+1 \quad b+2 \quad d+1 \\ 12 \dot{R} \dot{S} \dot{V} \\ a+2 \quad b+1 \quad d+1 \\ 12 \dot{R} \dot{S} \dot{V} \\ a+3 \quad b \quad d+1 \\ 4 \dot{R} \dot{S} \dot{V} \\ a \quad b+4 \quad d \\ 1 \dot{R} \dot{S} \dot{V} \\ a+1 \quad b+3 \quad d \\ 4 \dot{R} \dot{S} \dot{V} \\ a+2 \quad b+2 \quad d \\ 6 \dot{R} \dot{S} \dot{V} \\ a+3 \quad b+1 \quad d \\ 4 \dot{R} \dot{S} \dot{V} \\ a+4 \quad b \quad d \\ 1 \dot{R} \dot{S} \dot{V} \end{array} \right\} T^{c+m-4} + \&c.
 \end{aligned}$$

Aut etiam sequenti:

$$\begin{aligned}
 & \dot{R} \dot{T} \dot{V} \dot{S} + \frac{m}{1} \left\{ \begin{array}{l} \dot{R} \dot{T} \dot{V} \\ a \quad c \quad d+1 \\ \dot{R} \dot{T} \dot{V} \\ a \quad c+1 \quad d \\ \dot{R} \dot{T} \dot{V} \\ a+1 \quad c \quad d \\ \dot{R} \dot{T} \dot{V} \end{array} \right\} \dot{S}^{b+m-1} + \frac{m}{1} \times \frac{m-1}{2} \left\{ \begin{array}{l} 1 \dot{R} \dot{T} \dot{V} \\ a \quad c \quad d+2 \\ 2 \dot{R} \dot{T} \dot{V} \\ a \quad c+1 \quad d+1 \\ 2 \dot{R} \dot{T} \dot{V} \\ a+1 \quad c \quad d+1 \\ 1 \dot{R} \dot{T} \dot{V} \\ a \quad c+2 \quad d \\ 2 \dot{R} \dot{T} \dot{V} \\ a+1 \quad c+1 \quad d \\ 2 \dot{R} \dot{T} \dot{V} \\ a+2 \quad c \quad d \\ 1 \dot{R} \dot{T} \dot{V} \end{array} \right\} \dot{S}^{b+m-2} + \frac{m}{1} \times
 \end{aligned}$$

F 3

$$\begin{aligned}
 & + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \left\{ \begin{array}{l} \overset{a}{1} \overset{c}{R} \overset{d+3}{T} \overset{d+3}{V} \\ \overset{a}{3} \overset{c+1}{R} \overset{d+2}{T} \overset{d+2}{V} \\ \overset{a+1}{3} \overset{c}{R} \overset{d+2}{T} \overset{d+2}{V} \\ \overset{a}{3} \overset{c+2}{R} \overset{d+1}{T} \overset{d+1}{V} \\ \overset{a+1}{3} \overset{c+1}{R} \overset{d+1}{T} \overset{d+1}{V} \\ \overset{a+2}{6} \overset{c}{R} \overset{d+1}{T} \overset{d+1}{V} \\ \overset{a}{3} \overset{c+3}{R} \overset{d}{T} \overset{d}{V} \\ \overset{a+1}{1} \overset{c+2}{R} \overset{d}{T} \overset{d}{V} \\ \overset{a+2}{3} \overset{c+1}{R} \overset{d}{T} \overset{d}{V} \\ \overset{a+3}{3} \overset{c}{R} \overset{d}{T} \overset{d}{V} \\ \overset{a+3}{1} \overset{c}{R} \overset{d}{T} \overset{d}{V} \end{array} \right\} \overset{b+m-3}{S} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \left\{ \begin{array}{l} \overset{a}{1} \overset{c}{R} \overset{d+4}{T} \overset{d+4}{V} \\ \overset{a}{4} \overset{c+1}{R} \overset{d+3}{T} \overset{d+3}{V} \\ \overset{a+1}{4} \overset{c}{R} \overset{d+3}{T} \overset{d+3}{V} \\ \overset{a}{6} \overset{c+2}{R} \overset{d+2}{T} \overset{d+2}{V} \\ \overset{a+1}{6} \overset{c+1}{R} \overset{d+2}{T} \overset{d+2}{V} \\ \overset{a+2}{12} \overset{c}{R} \overset{d+2}{T} \overset{d+2}{V} \\ \overset{a}{6} \overset{c+3}{R} \overset{d+1}{T} \overset{d+1}{V} \\ \overset{a+1}{4} \overset{c+2}{R} \overset{d+1}{T} \overset{d+1}{V} \\ \overset{a+2}{12} \overset{c+1}{R} \overset{d+1}{T} \overset{d+1}{V} \\ \overset{a+3}{12} \overset{c}{R} \overset{d+1}{T} \overset{d+1}{V} \\ \overset{a}{4} \overset{c+4}{R} \overset{d}{T} \overset{d}{V} \\ \overset{a+1}{1} \overset{c+3}{R} \overset{d}{T} \overset{d}{V} \\ \overset{a+2}{4} \overset{c+2}{R} \overset{d}{T} \overset{d}{V} \\ \overset{a+3}{6} \overset{c+1}{R} \overset{d}{T} \overset{d}{V} \\ \overset{a+4}{4} \overset{c}{R} \overset{d}{T} \overset{d}{V} \\ \overset{a+4}{1} \overset{c}{R} \overset{d}{T} \overset{d}{V} \end{array} \right\} \overset{b+m-4}{S} + \&c.
 \end{aligned}$$

Vel denique huic:

$$\begin{aligned}
 & \overset{b}{S} \overset{c}{T} \overset{d}{V} \overset{a+m}{R} + \frac{m}{1} \left\{ \begin{array}{l} \overset{b}{S} \overset{c}{T} \overset{d+1}{V} \\ \overset{b}{S} \overset{c+1}{T} \overset{d}{V} \\ \overset{b+1}{S} \overset{c}{T} \overset{d}{V} \end{array} \right\} \overset{a+m-1}{R} + \frac{m}{1} \times \frac{m-1}{2} \left\{ \begin{array}{l} \overset{b}{2} \overset{c}{S} \overset{d+2}{T} \overset{d+2}{V} \\ \overset{b}{2} \overset{c+1}{S} \overset{d+1}{T} \overset{d+1}{V} \\ \overset{b+1}{2} \overset{c}{S} \overset{d+1}{T} \overset{d+1}{V} \\ \overset{b}{1} \overset{c+2}{S} \overset{d}{T} \overset{d}{V} \\ \overset{b+1}{2} \overset{c+1}{S} \overset{d}{T} \overset{d}{V} \\ \overset{b+2}{1} \overset{c}{S} \overset{d}{T} \overset{d}{V} \end{array} \right\} \overset{a+m-2}{R} + \frac{m}{1} \times
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \left\{ \begin{array}{l} 1 \dot{S} \dot{T} \dot{V} \\ b \quad c+1 \quad d+2 \\ 3 \dot{S} \dot{T} \dot{V} \\ b+1 \quad c \quad d+2 \\ 3 \dot{S} \dot{T} \dot{V} \\ b \quad c+2 \quad d+1 \\ 3 \dot{S} \dot{T} \dot{V} \\ b+1 \quad c+1 \quad d+1 \\ 6 \dot{S} \dot{T} \dot{V} \\ b+2 \quad c \quad d+1 \\ 3 \dot{S} \dot{T} \dot{V} \\ b \quad c+3 \quad d \\ 1 \dot{S} \dot{T} \dot{V} \\ b+1 \quad c+2 \quad d \\ 3 \dot{S} \dot{T} \dot{V} \\ b+2 \quad c+1 \quad d \\ 3 \dot{S} \dot{T} \dot{V} \\ b+3 \quad c \quad d \\ 1 \dot{S} \dot{T} \dot{V} \end{array} \right\} R + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \left\{ \begin{array}{l} 4 \dot{S} \dot{T} \dot{V} \\ b \quad c+1 \quad d+3 \\ 4 \dot{S} \dot{T} \dot{V} \\ b+1 \quad c \quad d+3 \\ 6 \dot{S} \dot{T} \dot{V} \\ b \quad c+2 \quad d+2 \\ 12 \dot{S} \dot{T} \dot{V} \\ b+1 \quad c+1 \quad d+2 \\ 6 \dot{S} \dot{T} \dot{V} \\ b+2 \quad c \quad d+2 \\ 4 \dot{S} \dot{T} \dot{V} \\ b \quad c+3 \quad d+1 \\ 12 \dot{S} \dot{T} \dot{V} \\ b+1 \quad c+2 \quad d+1 \\ 12 \dot{S} \dot{T} \dot{V} \\ b+2 \quad c+1 \quad d+1 \\ 4 \dot{S} \dot{T} \dot{V} \\ b+3 \quad c \quad d+1 \\ 1 \dot{S} \dot{T} \dot{V} \\ b \quad c+4 \quad d \\ 4 \dot{S} \dot{T} \dot{V} \\ b+1 \quad c+3 \quad d \\ 4 \dot{S} \dot{T} \dot{V} \\ b+2 \quad c+2 \quad d \\ 6 \dot{S} \dot{T} \dot{V} \\ b+3 \quad c+1 \quad d \\ 4 \dot{S} \dot{T} \dot{V} \\ b+4 \quad c \quad d \\ 1 \dot{S} \dot{T} \dot{V} \end{array} \right\} R + \&c.
 \end{aligned}$$

Quod ad legem progressionis in hisce seriebus attinet, iis id rursus commune est, quod eosdem coëfficientes, easdem uncias, eundemque etiam columnarum & singulorum terminorum numerum habeant. In prima columna iterum coëfficiens est unitas, in secunda $\frac{m}{1}$, in tertia $\frac{m}{1} \times \frac{m-1}{2}$, & sic deinceps, ac proinde in columna numero l erit coëfficiens $\frac{m}{1} \times \frac{m-1}{2} \times \&c. \dots \frac{m-l+2}{l-1}$. Porro in prima columna terminus est unus, in secunda tres videre est, in tertia sex, in quarta decem, in quinta quindecim, & ita porro: Itaque generaliter in columna l erit numerus terminorum $\frac{l}{1} \times \frac{l+1}{2}$. Quod ad uncias adinet, in prima columna nulla adest, in secunda uncia sunt 1, 1, 1, nempe coëffi-

coëfficientes trinomii simplicis, earumque summa est 3. In tertia vero 1, 2, 2, 1, 2, 1, videl. coëfficientes trinomii ad potestatem quadraticam evecti, qui sic formantur: Primo habes unitatem multiplicatam per primum coëfficientem potestatis secundæ, hoc est per unitatem; deinde duas unitates multiplicatas per secundum coëfficientem potestatis quadraticæ, hoc est per 2; tandem coëfficientes binomii quadratici, scilicet 1, 2, 1, multiplicatos per 1, nempe tertium & ultimum coëfficientem ejusdem potestatis: Summa igitur unciarum in hac tertia columna est 3×3 . Itaque generaliter in columna l uncia erunt coëfficientes trinomii ad potestatem $l-1$ evecti, & hi coëfficientes sic formabuntur: Prima uncia erit 1×1 ; deinde 1, 1, multiplicatæ per $\frac{l-1}{1}$; postmodum 1, 2, 1, (quæ sunt uncia potestatis quadraticæ) multiplicatæ per $\frac{l-1}{1} \times \frac{l-2}{2}$; tum 1, 3, 3, 1, (quæ sunt uncia tertiae potestatis) multiplicatæ per $\frac{l-1}{1} \times \frac{l-2}{2} \times \frac{l-3}{3}$; porro 1, 4, 6, 4, 1, (uncia scil. quartæ potestatis) per $\frac{l-1}{1} \times \frac{l-2}{2} \times \frac{l-3}{3} \times \frac{l-4}{4}$, & sic deinceps, idque donec perveniatur ad coëfficientes binomii potestatis $l-1$ multiplicatos per $\frac{l-1}{1} \times \frac{l-2}{2} \times \frac{l-3}{3} \times \dots \times \frac{1}{l-1}$, aut, quod idem est, per unitatem. Et generaliter summa unciarum in qualibet columna erit numerus ternarius ad dimensionem numero $l-1$ evectus. Quod vero ad indices fluxionales singulorum terminorum adtinet, dico quoad priorem seriem in quavis columna, puta l , singulos terminos in eadem rep ^{$d+m-l+1$} erandos habituros generalem multiplicatorem, scil. $\overset{a}{R} \overset{b}{S} \overset{c+l-1}{T}$, & quod ad ipsos terminos in qualibet columna cum dicta quantitate multiplicatos adtinet, primus eorum semper erit $\overset{a}{R} \overset{b}{S} \overset{c+l-1}{T}$; deinde duo erunt termini multiplicati per $\overset{c+l-2}{T}$, prior videl. $\overset{a}{R} \overset{b+1}{S}$, & alter, (ubi index quantitatis R unitate adcrefcit, dum index quantitatis S unitate decrefcit) $\overset{a+1}{R} \overset{b}{S}$; postea tres multiplicati per $\overset{c+l-3}{T}$, ubi iterum index ipsius R unitate augetur, dum index ipsius S unitate minuitur; deinde quatuor multiplicati per $\overset{c+l-4}{T}$, & sic porro; idque donec perveniatur ad $\overset{c}{T}$, ubi erunt termini numero l cum dicto $\overset{c}{T}$ multiplicati; & sic postremus

mus columnæ l terminus semper erit $\overset{a+l-1}{R} \overset{b}{S} \overset{c}{T}$. In secunda serie vero generalis omnium terminorum in quavis columna multiplicator est $\overset{c+m-l+1}{T}$, primusque terminus in eum ductus est $\overset{a}{R} \overset{b}{S} \overset{d+l-1}{V}$; deinde duo sequuntur termini cum $\overset{d+l-2}{V}$ multiplicati, quorum prior est $\overset{a}{R} \overset{b+1}{S}$, & alter, ubi index ipsius R unitate augetur, & ipsius S unitate minuitur, $\overset{a+1}{R} \overset{b}{S}$; postea tres termini multiplicati cum $\overset{d+l-3}{V}$, ubi itidem index ipsius R monade adcrefcit, dum index ipsius S eadem ratione decrefcit; deinde quatuor ducti in $\overset{d+l-4}{V}$, & sic porro; idque donec perveniatur ad $\overset{d}{V}$, ubi erunt termini numero l cum dicto $\overset{d}{V}$ multiplicati; sicque postremus columnæ illius terminus semper erit $\overset{a+l-1}{R} \overset{b}{S} \overset{d}{V}$. In tertia serie generalis multiplicator terminorum in qualibet columna est $\overset{b+m-l+1}{S}$, ubi circa quantitatem $\overset{c}{T}$ idem obtinet, ac in secunda serie circa quantitatem $\overset{b}{S}$; nam R & V in hac serie eodem loco collocatæ sunt, quam in secunda: Itaque prior terminus cum dicto $\overset{b+m-l+1}{S}$ multiplicatus semper erit $\overset{a}{R} \overset{c}{T} \overset{d+l-1}{V}$, ultimus vero $\overset{a+l-1}{R} \overset{c}{T} \overset{d}{V}$. In quarta serie generalis singularum columnarum multiplicator est $\overset{a+m-l+1}{R}$, ibique circa quantitatem $\overset{b}{S}$ idem obtinet, ac in tertia serie circa quantitatem $\overset{a}{R}$, nam T & V eodem loco hic collocatæ sunt, ac in tertia: Quare primus terminus cum dicto $\overset{a+m-l+1}{R}$ multiplicatus semper erit $\overset{b}{S} \overset{c}{T} \overset{d+l-1}{V}$, & ultimus $\overset{b+l-1}{S} \overset{c}{T} \overset{d}{V}$: Itaque in omnibus quatuor seriebus summa indicum fluxionalium ipsius $R S T V$ semper erit $a+b+c+d+m$.

Nunc iterum series has ad paucos casus particulares adplicaturi sumus, tam quoad fluxiones, quo casu quatuor hæ series

eandem plane rem producent, quam circa fluentes, ubi quatuor quidem prodibunt series, sed revera inter se æquales.

EXEMPLUM I.

Si investiganda fluxio prima ipsius $x y z u$, erit jam $R = x$, $S = y$, $T = z$, $V = u$, $a = b = c = d = 0$, $m = 1$; unde habebimus $x y z u + x y z u + x y z u + x y z u$. Quod si ponas $m = 2$, inveniemus ipsius $x y z u$ decrementum secundum æquale

$$x y z u + 2 \begin{Bmatrix} x y z \\ x y z \\ x y z \end{Bmatrix} u + 1 \begin{Bmatrix} 1 x y z \\ 2 x y z \\ 2 x y z \\ 1 x y z \\ 2 x y z \\ 1 x y z \end{Bmatrix} u$$

ubi omnes unciae cum coefficientibus simul sumptæ eadem sunt, quas reperires quadrinomial, hoc est summam quatuor quantitatum, ad potestatem quadraticam evehendo: Et sic ponendo $m = 3$, invenies coefficientes quadrinomiali ad potestatem cubicam perducti. In genere vero fluxio qualiscunque dictæ quantitatis $x y z u$ æquabitur

$$x y z u + \frac{m}{1} \begin{Bmatrix} x y z \\ x y z \\ x y z \end{Bmatrix} u + \frac{m-1}{1} \times \frac{m-1}{2} \begin{Bmatrix} 1 x y z \\ 2 x y z \\ 2 x y z \\ 1 x y z \\ 2 x y z \\ 1 x y z \end{Bmatrix} u + \&c.$$

EXEM-

E X E M P L U M II.

QUærat^r jam fluens quantitatis quadrimembris qualiscun-
que. Ita sit e. g. inveniendum incrementum primum
ipsius $x y z u$, erit jam rursus $R=x$, $S=y$, $T=z$, $V=u$,
 $a=b=c=d=0$; at $m=-1$; unde juxta priorem seriem desi-
deratum incrementum æquale erit

$$x y z u - \left\{ \begin{array}{c} x y z \\ x y z \\ x y z \end{array} \right\} u^{-2} + \left\{ \begin{array}{c} 1 x y z \\ 2 x y z \\ 2 x y z \\ 1 x y z \\ 2 x y z \\ 1 x y z \end{array} \right\} u^{-3} - \&c.$$

& juxta secundam

$$x y u z - \left\{ \begin{array}{c} x y u \\ x y u \\ x y u \end{array} \right\} z^{-2} + \left\{ \begin{array}{c} 1 x y u \\ 2 x y u \\ 2 x y u \\ 1 x y u \\ 2 x y u \\ 1 x y u \end{array} \right\} z^{-3} - \&c.$$

ac juxta tertiam

$$x z u y^{-1} - \left\{ \begin{matrix} x z u \\ x z u \\ x z u \end{matrix} \right\}^{-1} y^{-2} + \left\{ \begin{matrix} 1 x z u \\ 2 x z u \\ 2 x z u \\ 1 x z u \\ 2 x z u \\ 1 x z u \end{matrix} \right\}^{-2} y^{-3} - \&c.$$

denique juxta quartam

$$y z u x^{-1} - \left\{ \begin{matrix} y z u \\ y z u \\ y z u \end{matrix} \right\}^{-1} x^{-2} + \left\{ \begin{matrix} 1 y z u \\ 2 y z u \\ 2 y z u \\ 1 y z u \\ 2 y z u \\ 1 y z u \end{matrix} \right\}^{-2} x^{-3} - \&c.$$

Qua ratione vulgari methodo series hæ inveniri possent, ex iis, quæ in anterioribus Propositionibus diximus, non difficulter colligi poterit; quare molestum illum & nulli plane usui futurum laborem eam in rem impendere jam non lubet.

Similiter, ponendo $m = -2$, reperies itidem fluentem secundam ipsius $x y z u$ quatuor modis expressam: Et si quis in genere quaecunque tandem prædictæ quantitatis incrementum desideret, adhibeat duntaxat generales series sub initium hujus Propositionis exhibitas, ibique pro R substituatur x , item y pro S , ac z loco T , denique u pro V , & ponat a, b, c, d nihilo æquales, & sic quatuor seriebus fluentem cujuscunque ordinis indagabit.

EXEM.

E X E M P L U M I I I.

SI reversio datæ fluxionis vel fluentis ex nostra serie sit expiscanda, capiat duntaxat ipsius $\overset{m}{x} \overset{a}{y} \overset{b}{z} \overset{c}{u}$ (quam initio hujus Propositionis in infinita serie quatuor modis exhibuimus,)

rursus varietatem ordinis p , & habebit $\overset{m+p}{x} \overset{a}{y} \overset{b}{z} \overset{c}{u}$ itidem quatuor seriebus expressam; deinde ex coefficientibus datæ fluxionis vel fluentis videat, quis sit valor numeri m ; item ex datis indicibus fluxionalibus comparatis cum iis, quas in infinitis nostris seriebus cernere est, videat, qui sint valores indicum a, b, c, d ; deinde ponat $m+p$ æquale nihilo, & inveniet quantitatem quæsitam. Sic etiam, si datæ fluxionis vel fluentis rursus decrementum vel incrementum aliquod investigandum sit, videat itidem, quis sit valor numeri m , & indicum a, b, c, d ; deinde p æqualem ponat tali numero, qualis est ordo novæ fluxionis vel fluentis capiendæ; sicque $m+p$ exprimet genus decrementi vel incrementi ipsius quantitatis $\overset{a}{x} \overset{b}{y} \overset{c}{z} \overset{d}{u}$. Possemus hæc latius exsequi, nisi ea ex præcedentibus Propositionibus unicuivis manifesta fore arbitraremur.

E X E M P L U M I V.

SI quis ope harum serierum invenire velit incrementum vel decrementum quantitatis alicujus quatuor dimensionum, puta $\overset{a}{x} \overset{a}{x} \overset{a}{x} \overset{a}{x}$, hoc est $\overset{a}{x} \overset{a}{x} \overset{a}{x} \overset{a}{x}$, tum erit $R=S=T=V=x$, & $a=b=c=d$; ideoque erit fluens vel fluxio qualiscunque

$$\overset{a}{x} \overset{a}{x} \overset{a}{x} + \frac{m}{1} \times 3 \overset{a}{x} \overset{a}{x} \overset{a}{x} + \frac{m}{1} \times \frac{m-1}{2} \left\{ \begin{array}{l} 3 \overset{a}{x} \overset{a}{x} \\ 6 \overset{a}{x} \overset{a}{x} \end{array} \right\} + \frac{m}{1} \times$$

$$+ \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \left\{ \begin{array}{c} \frac{a^{(2)}}{3} \frac{a+3}{x} \\ 18 \frac{a}{x} \frac{a+2}{x} \frac{a+1}{x} \\ \frac{a^{(3)}}{6} x \end{array} \right\} x^{a+m-3} + \&c.$$

& sic una tantum prodit series, tam respectu fluentium, quam fluxionum.

E X E M P L U M V.

Quod si ope seriei nostræ investiganda sit fluens $4 x^{\frac{1}{2}} x$, sive $4 x x x x$, erit $R=S=T=V=x$, $a=b=c=0$, $d=1$, & $m=-1$, unde juxta primam seriem fluens quæsitæ æquabitur

$$4 x^{\frac{1}{2}} - 12 x^{\frac{1}{2}} x^{\frac{1}{2}} + \left\{ \begin{array}{c} 12 x^{\frac{1}{2}} x^{\frac{1}{2}} \\ 24 x x \end{array} \right\} x^{\frac{1}{2}} - \left\{ \begin{array}{c} 12 x^{\frac{1}{2}} x^{\frac{1}{2}} \\ 72 x x x \\ 24 x^{\frac{1}{2}} \end{array} \right\} x^{\frac{1}{2}} + \&c.$$

& juxta secundam, tertiam, & quartam,

$$4 x^{\frac{1}{2}} x^{\frac{1}{2}} - \left\{ \begin{array}{c} 4 x^{\frac{1}{2}} x^{\frac{1}{2}} \\ 8 x x \end{array} \right\} x^{\frac{1}{2}} + \left\{ \begin{array}{c} 4 x^{\frac{1}{2}} x^{\frac{1}{2}} \\ 24 x x x \\ 8 x^{\frac{1}{2}} \end{array} \right\} x^{\frac{1}{2}} - \&c.$$

adeoque, quum hæ duæ series æquales sint, adæquando eas, & transponendo, ac deinde per 4 dividendo, fiet

$$x^{\frac{1}{2}} = 4 x^{\frac{1}{2}} x^{\frac{1}{2}} - \left\{ \begin{array}{c} 4 x^{\frac{1}{2}} x^{\frac{1}{2}} \\ 8 x x \end{array} \right\} x^{\frac{1}{2}} + \left\{ \begin{array}{c} 4 x^{\frac{1}{2}} x^{\frac{1}{2}} \\ 24 x x x \\ 8 x^{\frac{1}{2}} \end{array} \right\} x^{\frac{1}{2}} - \&c.$$

quam

quam seriem infinitam si compendii gratia vocemus Q , habebimus juxta primam seriem ipsius $4 \bar{x}^{(3)} \dot{x}$ fluentem primam æqualem $4 \bar{x}^{(4)} - 3 Q$, hoc est, (quoniam jam Q ipsi $\bar{x}^{(4)}$ æquatur,) $4 \bar{x}^{(4)} - 3 \bar{x}^{(4)}$, sive $\bar{x}^{(4)}$; & juxta secundam, tertiam, & quartam statim habebimus Q , adeoque $\bar{x}^{(4)}$, quod proinde ex serie nostra deducimus esse incrementum primum ipsius $4 \bar{x}^{(3)} \dot{x}$, id quod cum veritate omni ex parte consentaneum est.

E X E M P L U M V I.

QUod si quis in universali nostra serie pro quantitate R , S , T , vel V adhibere velit quantitatem constantem, pone n , tum tres duntaxat prodibunt series; quum ea, quæ alioquin incrementa & decrementa ipsius n exhiberet, locum jam non inveniat, quoniam n est quantitas invariabilis. Ita ad invenien-
dam qualemcunque sive fluxionem sive fluentem ipsius $n \overset{a}{x} \overset{b}{y} \overset{c}{z}$, erit $R=x$, $S=y$, $T=z$, $V=n$, $d=0$; sicque incidet in tres series anterioris Propositionis multiplicatas per n , & posito $n=1$, eadem plane prodibunt series. Quod si adhuc aliam quantitatem constantem ponas, e. g. k , ita ut varietatem desideres ipsius $k n \overset{a}{x} \overset{b}{y}$, erit $R=x$, $S=y$, $T=k$, $V=n$, $c=0$, $d=0$, tum duas habebis series, quæ eodem incident, ac eæ, quas dedi in principio *Proposit. I*, & ponendo $k n=1$, ipsissimæ illæ prodibunt series: Ut adeo jam pateat, quo plures adhibeantur quantitates in semet multiplicatæ, quarum qualiscunque tandem varietas quæritur, eo universalius esse Problema, quum plures illæ semper reduci queant ad pauciores.



P R O P O S I T I O I V .

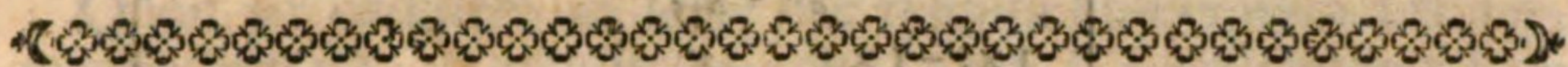
*Generalem Regulam, qua cujuscunque ordinis fluxio vel fluens
quotlibet quantitatum variabilium in semet multipli-
catum inveniri potest, præscribere.*

DEtur jam productum ex quocunque quantitibus fluxio-
nalibus, numero r , in semet multiplicatis constans,
quod jam sic exprimo, $\overset{a}{A} \times \overset{b}{B} \times \&c. \times \overset{x}{X} \times \overset{y}{Y} \times \overset{z}{Z}$, ubi tantum duas
prioris & tres ultimas quantitates adpono. Ejus jam desidere-
tur sive fluxio sive fluens ordinis (m): Clarum est ex præce-
dentibus Propositionibus, primam columnam constitutam ex
uno termino, qui erit $\overset{a}{A} \times \overset{b}{B} \times \&c. \times \overset{x}{X} \times \overset{y}{Y} \times \overset{z+m}{Z}$; deinde sequitur
columna secunda, ubi generalis omnium terminorum multipli-
cator erit $\overset{z+m-1}{Z}$, eruntque in ea termini numero $r-1$, quorum
primus cum dicto $\overset{z+m-1}{Z}$ multiplicatus erit $\overset{a}{A} \times \overset{b}{B} \times \&c. \times \overset{x}{X} \times \overset{y+1}{Y}$,
secundus $\overset{a}{A} \times \overset{b}{B} \times \&c. \times \overset{x+1}{X} \times \overset{y}{Y}$; deinde sequuntur plures termini,
ubi continuo alii indici fluxionali unitas addenda erit, idque
juxta ordinem, quem obtinent quantitates in dato producto;
& sic erit penultimus terminus $\overset{a}{A} \times \overset{b+1}{B} \times \&c. \times \overset{x}{X} \times \overset{y}{Y}$, ultimus
vero $\overset{a+1}{A} \times \overset{b}{B} \times \&c. \times \overset{x}{X} \times \overset{y}{Y}$: Itaque in columna secunda prædicta,
ubi generalis multiplicator, ut jam diximus, est $\overset{z+m-1}{Z}$, erit unus
terminus ductus in $\overset{y+1}{Y}$, & $r-2$ termini in $\overset{y}{Y}$. Jam succedit
columna tertia, ubi generalis omnium terminorum multiplica-
tor est $\overset{z+m-2}{Z}$, eruntque in ea termini numero $\frac{r-1}{1} \times \frac{r}{2}$, quorum pri-
mus cum dicto $\overset{z+m-2}{Z}$ multiplicatus erit $\overset{a}{A} \times \overset{b}{B} \times \&c. \times \overset{x}{X} \times \overset{y+2}{Y}$, se-
cundus $\overset{a}{A} \times \overset{b}{B} \times \&c. \times \overset{x+1}{X} \times \overset{y+1}{Y}$: Deinde erunt plures termini, ubi
indi-

indices fluxionales continuo mutabuntur, idque tot vicibus,
 quot poterunt: Sicque erit penultimus terminus $\overset{a+1}{A} \times \overset{b+1}{B} \times \&c.$
 $\times \overset{x}{X} \times \overset{y}{Y}$, & ultimus $\overset{a+2}{A} \times \overset{b}{B} \times \&c. \times \overset{x}{X} \times \overset{y}{Y}$: Quare in dicta co-
 lumna tertia, quæ generalem multiplicatorem habet $\overset{z+m-2}{Z}$, erit
 unus terminus multiplicatus cum $\overset{y+2}{Y}$, & $r-2$ termini cum $\overset{y+1}{Y}$, ac
 $\frac{r-2}{1} \times \frac{r-1}{2}$ termini cum $\overset{y}{Y}$. Summa indicum fluxionalium ipsius
 $\overset{a}{A} \times \overset{b}{B} \times \&c. \times \overset{x}{X} \times \overset{y}{Y} \times \overset{z}{Z}$ & in hac & in omnibus aliis columnis
 semper erit $a+b+\&c. +x+y+z+m$. Unciæ erunt coëffi-
 cientes $r-1$ nomii (si ita loqui licet) ad potestatem quadrati-
 cam evecti, earumque summa $\overline{r-1}^{(2)}$. Jam vero generaliter in
 quavis columna, puta l , numerus terminorum, quorum gene-
 ralis multiplicator jam est $\overset{z+m-l+1}{Z}$, erit $\frac{r-1}{1} \times \frac{r}{2} \times \frac{r+1}{3} \times \&c. \dots \frac{r+l-3}{l-1}$,
 aut quod eodem redit, & per quod res in dato casu singulari
 citius expediri potest, $\frac{l}{1} \times \frac{l+1}{2} \times \frac{l+2}{3} \times \&c. \dots \frac{l+r-3}{r-2}$, inque prædicta
 columna præter generalem multiplicatorem unus est terminus
 ductus in $\overset{y+l-1}{Y}$, deinde $\frac{r-2}{1}$ termini in $\overset{y+l-2}{Y}$, postea $\frac{r-2}{1} \times \frac{r-1}{2}$ termi-
 ni in $\overset{y+l-3}{Y}$, tum $\frac{r-2}{1} \times \frac{r-1}{2} \times \frac{r}{3}$ termini in $\overset{y+l-4}{Y}$, & sic porro; id-
 que donec perveniatur ad $\overset{y}{Y}$, cum quo multiplicati erunt ter-
 mini numero $\frac{r-2}{1} \times \frac{r-1}{2} \times \frac{r}{3} \times \&c. \dots \frac{r+l-4}{l-1}$, aut quod idem est,
 & brevius procedit, $\frac{l}{1} \times \frac{l+1}{2} \times \frac{l+2}{3} \times \&c. \dots \frac{l+r-4}{r-3}$: Itaque primus
 terminus cum dicto $\overset{z+m-l+1}{Z}$ multiplicatus semper erit $\overset{a}{A} \times \overset{b}{B} \times \&c.$
 $\times \overset{x}{X} \times \overset{y+l-1}{Y}$, secundus vero $\overset{a}{A} \times \overset{b}{B} \times \&c. \times \overset{x+1}{X} \times \overset{y+l-2}{Y}$; & penulti-
 mus $\overset{a+l-2}{A} \times \overset{b+1}{B} \times \&c. \times \overset{x}{X} \times \overset{y}{Y}$, denique ultimus erit $\overset{a+l-1}{A} \times \overset{b}{B} \times \&c.$
 $\times \overset{x}{X} \times \overset{y}{Y}$. Unciæ hic erunt coëfficientes $\overline{r-1}$ nomii ad potesta-
 tem $l-1$ evecti, earumque summa $\overline{r-1}^{(l-1)}$. Hæc de prima serie
 dicta sunt. In secunda generalis omnium terminorum in qua-

vis columna, puta l , multiplicator est $\dot{Y}^{y+m-l+1}$; circa uncias vero & numerum terminorum idem in omnibus, ac in præcedenti obtinet. In tertia universalis multiplicator est $\dot{X}^{s+m-l+1}$, & sic porro. In penultima $\dot{B}^{b+m-l+1}$. Et in postrema $\dot{A}^{a+m-l+1}$: Quare hic habes tot series, quot reperiuntur in dato producto quantitates, adeoque numero r , quæ, ubi m est numerus positivus seu affirmativus, eandem repræsentabunt fluxionem; ast ubi privativus seu negativus, r modis fluens exhibebitur: Quod si tamen aliquæ quantitates ex his ponantur æquales, tum etiam pauciores prodibunt series; modo tamen & indices fluxionales earum quantitatum in hoc casu sibi adæquentur: e. g. si inter quantitates numero r ponas aliquot numero s aliis inter se æquales, habebis $r-s$ series: Sic si dato $\dot{A}^a \times \dot{B}^b \times \&c. \times \dot{X}^s \times \dot{Y}^y \times \dot{Z}^z$, poneres unam alteri æqualem, pone $\dot{Y}^y = \dot{Z}^z$, ita ut haberes $\dot{A}^a \times \dot{B}^b \times \&c. \times \dot{X}^s \times \dot{Y}^{y(2)}$, jam prodirent $r-1$ series, & sic porro: Et ponendo $r-3$ æquales, habebis tres series, & sic deinceps. Idemque obtinet, si aliquot quantitates constantes ponas; nam ponendo s quantitates constantes, habebis $r-s$ series. Ita ponendo $r-4$ constantes, prodibunt quatuor series, & in illo casu incidēs in *Proposit. III*; si $r-3$ constantes, tres invenientur series, tumque idem erit casus, qui est in *Proposit. II*: Si $r-2$ invariabiles, duas duntaxat series reperturus es, & invenies *Proposit. I*, ubi pro quantitatibus illis constantibus unitatem adhibueris.





PROPOSITIO V.

*Quantitatis variabilis ad quocunque dimensiones elatae fluxiones
& fluentes cujuscunque ordinis in serie infinita exhibere.*

Si quantitas $\dot{x}$ evecta ad r dimensiones, ejusque desideretur
sive decrementum sive incrementum ordinis (m), di-
co $\dot{x}$ æquale fore sequenti seriei infinitæ:

$$\begin{aligned} & \frac{\dot{x}^{a+m}}{\dot{x}^{a+r-1}} + \frac{m}{1} \times \frac{\dot{x}^{a+m-1}}{\dot{x}^{a+r-2}} + \frac{m(m-1)}{1 \times 2} \times \frac{\dot{x}^{a+m-2}}{\dot{x}^{a+r-3}} + \frac{m(m-1)(m-2)}{1 \times 2 \times 3} \times \frac{\dot{x}^{a+m-3}}{\dot{x}^{a+r-4}} + \dots \\ & \left\{ \begin{array}{l} \frac{\dot{x}^{a+m-2}}{\dot{x}^{a+r-1} \times \dot{x}^{a+2}} \\ \frac{\dot{x}^{a+m-3}}{\dot{x}^{a+r-1} \times \dot{x}^{a+1} \times \dot{x}^{a+1}} \end{array} \right\} \\ & + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{\dot{x}^{a+m-3}}{\dot{x}^{a+r-1} \times \dot{x}^{a+3}} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \times \frac{\dot{x}^{a+m-4}}{\dot{x}^{a+r-1} \times \dot{x}^{a+2} \times \dot{x}^{a+1} \times \dot{x}^{a+1}} + \dots \end{aligned}$$

Quod ad ordinem in serie hac infinita observatum adtinet, manifestum est, in quavis columna, puta l , coëfficientem fore $\frac{m}{1} \times \frac{m-1}{2} \times \&c. \dots \frac{m-l+2}{l-1}$, & generalem omnium terminorum, qui

in dicta columna reperiuntur, multiplicatorem $x^{a+m-l+1}$: Primusque in ea terminus, incipiendo a columna secunda, erit $r-1$

$\times x^{\frac{a(r-2)}{a+l-1}}$ x : Deinde index fluxionalis $a+l-1$ scinditur in tot partes, & tot vicibus, quot potest; idque ita, ut si partes rescissæ dentur numero s , etiam multiplicatores futuri sint numero s ,

(non computatis jam unciis, vel quantitate $x^{\frac{a(r-s-1)}{a}}$,) qui incipiunt ab $r-1$, & deinde continuo unitate decrescunt, usque dum perveniatur ad $r-s$. Ita e. g. in columna quinta index $a+4$ potest scindi in duas partes, $a+3$, $a+1$; quare termi-

nus ille erit $r-1 \times r-2 \times x^{\frac{a(r-3)}{a+3}} x^{\frac{a+1}{a+1}}$ (non computatis jam, ut modo dictum, unciis;) Porro adhuc scindi potest in duas partes, videl. $a+2$, $a+2$, aut, quod idem est, $\overline{a+2}^{(2)}$; unde

terminus ille erit $r-1 \times r-2 \times x^{\frac{a(r-3)}{a+2}} x^{\frac{a+2}{a+2}}$. Tum $a+4$ in tres partes secari potest, nempe $a+2$, $a+1$, $a+1$, sive $a+2$, $\overline{a+1}^{(2)}$; ficque terminus ille erit $r-1 \times r-2 \times r-3 \times x^{\frac{a(r-4)}{a+2}} x^{\frac{a+2}{a+2}} x^{\frac{a+1}{a+1}}$; denique

in quatuor partes secari potest, scil. $a+1$, $a+1$, $a+1$, $a+1$, sive $\overline{a+1}^{(4)}$; eritque proinde ultimus in dicta columna quinta terminus $r-1 \times r-2 \times r-3 \times r-4 \times x^{\frac{a(r-5)}{a+1}} x^{\frac{a+1}{a+1}}$. Erit itaque in quali-

bet columna, puta l , primus terminus $r-1 \times x^{\frac{a(r-2)}{a+l-1}}$ (modo tamen l sit unitate major, nam in prima columna non datur

multiplicator $r-1$,) secundus $r-1 \times r-2 \times x^{\frac{a(r-3)}{a+l-2}} x^{\frac{a+1}{a+1}}$ (modo tum l sit major numero binario, quippe nec in columna prima nec in secunda reperitur multiplicator $r-1 \times r-2$,) nulla jam unciarum, quas terminus ille in singulis columnis diversas præ-

positas habet, ratione habita: Penultimus itidem sine respectu unciarum erit $r-1 \times r-2 \times \&c. \dots r-l+2 \times x$ $\frac{1}{a^{(r-l+1)}} \frac{1}{a+2} \frac{1}{a+1}^{(l-3)}$ x x (dummodo & hic numerus l binarium superet,) denique ultimus $r-1 \times r-2 \times \&c. \dots r-l+1 \times x$ $\frac{1}{a^{(r-l)}} \frac{1}{a+1}^{(l-1)}$ x , qui etiam in columna prima procedit, quoniam ibi multiplicator $1 \times r-1 \times r-2 \times \&c. \dots r-l+1$ æqualis est unitati. Et ex hisce quidem abunde, ut puto, de natura hujus seriei sic satis intricatæ eatenus constat: At longe adhuc major difficultas est in detegendis unciis, quas singuli termini præfixi habent: Primam quidem & ultimam video in omnibus columnis esse unitatem. Secundam dices, incipiendo a columna quarta, esse $l-1$, & penultimam ubique $\frac{l-1}{1} \times \frac{l-2}{2}$. De reliquis vero alii me sagaciores viderint; ego etenim, etsi multum in id inpendere laboris, earum ordinem indagare hucusque haud potis fui; quamvis progressionem aliquam iis inesse nullus dubitem. Hoc tantum monebo, in quavis columna summam multiplicantium illorum $r-1$, $r-1 \times r-2$, &c. computatis unciis, fore $r-1^{(l-1)}$, juxta ea, quæ in præcedenti Propositione vidimus.

E X E M P L U M I.

POnatur exempli causa $m=1$; erit igitur fluxio prima ipsius x $\frac{1}{a^{(r)}} \frac{1}{a+1}$ juxta seriem nostram infinitam æqualis x $\frac{1}{a^{(r-1)}} \frac{1}{a+1}$ $x + r-1 \times x$ $\frac{1}{a^{(r-2)}} \frac{1}{a+1} \frac{1}{a}$ x , aut quod idem est, $r \times x$ $\frac{1}{a^{(r-1)}} \frac{1}{a+1}$ x . Ita posito $m=2$, invenietur secunda fluxio x $\frac{1}{a^{(r-1)}} \frac{1}{a+2}$ $x + 2 \times r-1 \times x$ $\frac{1}{a^{(r-2)}} \frac{1}{a+1}^{(2)}$ $x + r-1 \times x$ $\frac{1}{a^{(r-2)}} \frac{1}{a+1}^{(2)}$ x $\frac{1}{a^{(r-3)}} \frac{1}{a+1}^{(2)}$ x , sive $r \times x$ $\frac{1}{a^{(r-1)}} \frac{1}{a+2}$ $x + r \times r-1 \times x$ $\frac{1}{a^{(r-2)}} \frac{1}{a+1}^{(2)}$ x . Similiter, posito $m=3$, erit ipsius x $\frac{1}{a^{(r)}} \frac{1}{a+1} \frac{1}{a+2}$ fluxio tertia æqualis $r \times x$ $\frac{1}{a^{(r-1)}} \frac{1}{a+1} \frac{1}{a+2}$

$\frac{r \times x}{a+1} + \frac{3 \times r \times r - 1 \times x}{a+2} + \frac{r \times r - 1 \times r - 2 \times x}{a+3} + \frac{4 \times r \times r - 1 \times x}{a+4} + \frac{3 \times r \times r - 1 \times x}{a+5} + \frac{6 \times r \times r - 1 \times r - 2 \times x}{a+6} + \frac{r \times r - 1 \times r - 2 \times r - 3 \times x}{a+7}$, & sic deinceps: Unde videmus, fluxionem primam secundæ columnæ, secundam tertię, tertiam quartæ, quartam fluxionem quintæ columnæ correspondere: Itaque generaliter fluxio ordinis (m) congruet cum terminis columnæ $m+1$, dummodo hoc observetur, ut in multiplicatoribus terminorum pro $r-1$ scribatur r , pro $r-2$ scribatur $r-1$, & sic porro, semper addendo unitatem, ideoque pro $r-s$ reponendo $r-s+1$: Unde videmus, quam facili negotio ope hujus seriei fluxio cujuscunque gradus detegi possit, idque auxilio unius columnæ: Sic si quæreretur ipsius x fluxio sexta, adhiberi poterit brevitatis causa duntaxat columna septima neglectis sex prioribus; eritque proinde illius fluxionis primus terminus $r \times x$, secundus $6 \times r \times r - 1 \times x$, tertius $15 \times r \times r - 1 \times x$, & sic de reliquis. Poterit vero hæc series etiam ad fluentes applicari, ubi m est numerus negativus, sicque semper series remanebit infinita; sed hoc curioso Lectori relinquimus.

E X E M P L U M I I.

JAm si retineatur numerus m ut indeterminatus, at pro r substituatur numerus quicunque, tum series quidem erit infinita; sed tum in columna $r+1$ & sequentibus aliqui termini excident, si scil. r sit numerus integer adfirmativus, & unitate

te major; nam si r ponatur æqualis 1, habebimus duntaxat $\frac{1}{a^{(0)}} \dot{x}^{a+m}$, hoc est $1 \times \dot{x}^{a+m}$. Itaque si fiat $r=2$, erit ipsius $\dot{x}^{a+m}$ fluxio & fluens ordinis (m) æqualis $\frac{a}{a+1} \dot{x}^{a+m} + \frac{m}{1} \times \frac{a+1}{a+2} \dot{x}^{a+m-1} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{a+2}{a+3} \dot{x}^{a+m-2} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{a+3}{a+4} \dot{x}^{a+m-3} + \&c.$ convenienter iis, quæ in *Proposit. I. Exempl. 19.* vidimus.

Posito vero $r=3$, erit quantitatis $\dot{x}^{a+m}$ incrementum vel decrementum generis (m) æquale huic seriei:

$$\frac{a}{a+1} \dot{x}^{a+m} + \frac{m}{1} \times 2 \times \frac{a}{a+2} \dot{x}^{a+m-1} + \frac{m}{1} \times \frac{m-1}{2} \left\{ \begin{array}{l} 2 \times \frac{a}{a+3} \dot{x}^{a+m-2} \\ 2 \times \frac{a+1}{a+2} \dot{x}^{a+m-2} \end{array} \right\} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \left\{ \begin{array}{l} 2 \times \frac{a}{a+4} \dot{x}^{a+m-3} \\ 6 \times \frac{a+1}{a+3} \dot{x}^{a+m-3} \end{array} \right\} + \&c.$$

quæ omnia conveniunt cum iis, quæ in *Proposit. II. Exempl. 9.* deteximus.

Ita etiam si ponatur $r=4$, erit ipsius $\dot{x}^{a+m}$ varietas ordinis (m) æqualis sequenti seriei infinitæ:

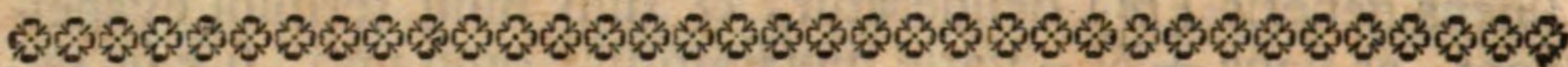
$$\frac{a}{a+1} \dot{x}^{a+m} + \frac{m}{1} \times 3 \times \frac{a}{a+2} \dot{x}^{a+m-1} + \frac{m}{1} \times \frac{m-1}{2} \left\{ \begin{array}{l} 3 \times \frac{a}{a+3} \dot{x}^{a+m-2} \\ 6 \times \frac{a+1}{a+2} \dot{x}^{a+m-2} \end{array} \right\} + \&c.$$

congruenter iis, quæ *Proposit. III. Exempl. 4.* invenimus.

Imo etiam series hæc procedit, ubi r est numerus negativus, adeoque ubi ipsius $\frac{1}{\dot{x}}$, vel $\frac{1}{\dot{x} \dot{x}}$, aut $\frac{1}{\dot{x} \dot{x} \dot{x}}$, &c. fluxio vel fluens

fluens generis (m) desideratur, & tum nulli termini evanescent, at signa $+$ & $-$ varie mutabuntur; quin imo, si r sit numerus fractus sive positivus sive negativus, quo casu itidem omnes termini remanebunt; videlicet, si, ut jam dixi, pro indice ordinis fluxionalis universaliter adhibeatur quantitas m ; nam alioqui si pro m substituatur aliquis numerus integer affirmativus, semper, ut modo vidimus, termini evaderent numero finiti, qualiscunque tandem esset valor ipsius r .





P R O P O S I T I O VI.

Quantitatem variabilem, sive puram, sive fluxionalem, & quotcunque dimensionum seriei infinitæ fluxionali adæquare.

D Etur quantitas $\dot{x}$, proinde variabilis, adeoque vel pura, ubi $a=0$, vel fluxionalis, ubi a qualicunque tandem numero æqualis est; sitque prædicta quantitas elevata ad potestatem ordinis (r) , & quærat ^{$\frac{a}{a+1}$} ur ejus valor in serie fluxionali, dico ipsum $\dot{x}$ æquale fore sequenti seriei:

$$\begin{aligned}
 & r \times \dot{x} \quad \dot{x} \quad \dot{x} - \left\{ \begin{array}{cc} \frac{\dot{a}^{(r-2)} \dot{a}+2}{r \times \dot{x}} & \dot{x} \\ \frac{\dot{a}^{(r-3)} \dot{a}+1(2)}{r \times r - 2 \times \dot{x}} & \dot{x} \end{array} \right\} \dot{x} + \left\{ \begin{array}{cc} \frac{\dot{a}^{(r-2)} \dot{a}+3}{1 \times r \times \dot{x}} & \dot{x} \\ \frac{\dot{a}^{(r-3)} \dot{a}+2 \dot{a}+1}{3 \times r \times r - 2 \times \dot{x}} & \dot{x} \quad \dot{x} \\ \frac{\dot{a}^{(r-4)} \dot{a}+1(3)}{1 \times r \times r - 2 \times r - 3 \times \dot{x}} & \dot{x} \end{array} \right\} \dot{x} \\
 & - \left\{ \begin{array}{cc} \frac{\dot{a}^{(r-2)} \dot{a}+4}{1 \times r \times \dot{x}} & \dot{x} \\ \frac{\dot{a}^{(r-3)} \dot{a}+3 \dot{a}+1}{4 \times r \times r - 2 \times \dot{x}} & \dot{x} \quad \dot{x} \\ \frac{\dot{a}^{(r-3)} \dot{a}+2(2)}{3 \times r \times r - 2 \times \dot{x}} & \dot{x} \\ \frac{\dot{a}^{(r-4)} \dot{a}+2 \dot{a}+1(2)}{6 \times r \times r - 2 \times r - 3 \times \dot{x}} & \dot{x} \quad \dot{x} \\ \frac{\dot{a}^{(r-5)} \dot{a}+1(4)}{1 \times r \times r - 2 \times r - 3 \times r - 4 \times \dot{x}} & \dot{x} \end{array} \right\} \dot{x} + \&c.
 \end{aligned}$$

Ordinem hujus seriei si quis quærat, conferat eam cum illa, quam in *Proposit. V.* exhibui, & notet has differentias. Quæ ibi columna prima est, ea hic desideratur, ideoque columna
prima

prima in hac serie conferenda est cum secunda in anteriori serie, & generaliter columna l in hac serie conferenda est cum columna $l+1$ in anteriori. Porro coëfficientes $\frac{m}{1}, \frac{m}{1} \times \frac{m-1}{2}, \&c.$ qui ibi adsunt, hic non reperiuntur; verum singulis columnis hic alternatim diversum signum præpositum est, nunc $+$, modo $-$. Insuper quantitas $r-1$, cum qua omnes termini in quavis columna (dempta prima) in præcedenti serie multiplicati sunt, hic non adest; sed ejus loco reperitur quantitas r , idque in omnibus cujusvis columnæ terminis. Denique generalis multiplicator, qui ibi pro qualibet columna, puta $l+1$, est

$\frac{a+m-l}{a-l}$ x , ea hic pro columna l est x ; idque, quoniam hic non desideratur fluxio vel fluens ordinis (m), ideoque numerus m in hac serie reperiri non potest: Circa reliqua autem omnia, videl. circa uncias, multiplicatores $r-2, r-3, \&c.$ ipsasque quantitates cæteras fluxionales idem obtinet in hac atque in priori serie; unde, qui illam recte intelligit, etiam hujus ordinem facile adsequetur, & proinde eam continuare absque ullo negotio poterit. Quod autem ad veritatem hujus Propositionis

adtinet, quod scil. $\frac{a}{a-l} x^{(r)}$ præfatæ seriei æqualis sit, ea sponte

sua elucebit, ubi & quantitatis $\frac{a}{a-l} x^{(r)}$ & ipsius infinitæ seriei fluxio-

nem primam ceperis; sic enim habebimus $r \times \frac{a}{a-l} x^{(r-1)} = r \times \frac{a}{a-l} x^{(r-2)}$

$\frac{a}{a-l} x^{(r-1)}$ x , (reliqui enim omnes termini seriei fluxiones capiendo continuo sese mutuo destruent,) id quod nemo negabit; jam vero, quum fluxiones hæ sint æquales, erunt etiam earum incrementa, adeoque quantitates ipsæ inter se æquales. Nunc vero seriem hancce ad nonnullos casus particulares adplicabimus, ubi $a=0$, adeoque ubi quantitas x est pura, & tamen nihilominus variabilis; (si enim quantitas fluxionalis foret, nemo forte inficias iret, eam in seriem infinitam fluxionalem resolvi posse.)

E X E M P L U M . I.

POnatur jam exempli causa $r=2$, & (prout modo dictum,) $a=0$, erit juxta seriem hanc $\bar{x}^{-(2)} = 2 \bar{x}^{\frac{1}{1} \frac{-1}{1}} - 2 \bar{x}^{\frac{2}{2} \frac{-2}{2}} + 2 \bar{x}^{\frac{3}{3} \frac{-3}{3}} - 2 \bar{x}^{\frac{4}{4} \frac{-4}{4}} + \&c.$ perinde, ac in *Proposit. I. Exempl. 20.* vidimus. Quod si ponas $r=3$, habebimus.

$$\bar{x}^{-(3)} = 3 \bar{x}^{\frac{1}{1} \frac{-1}{1}} - \left\{ \begin{array}{c} 3 \bar{x}^{\frac{2}{2} \frac{-2}{2}} \\ \frac{1}{1}^{(2)} \end{array} \right\} \bar{x} + \left\{ \begin{array}{c} 3 \bar{x}^{\frac{3}{3} \frac{-3}{3}} \\ 9 \bar{x}^{\frac{2}{2} \frac{-1}{1}} \end{array} \right\} \bar{x} - \left\{ \begin{array}{c} 3 \bar{x}^{\frac{4}{4} \frac{-4}{4}} \\ 12 \bar{x}^{\frac{3}{3} \frac{-2}{2}} \\ 9 \bar{x}^{\frac{2}{2} \frac{-2}{2}} \end{array} \right\} \bar{x} + \&c.$$

congruenter iis, quæ in *Proposit. II. Exempl. 10.* diximus.

Ita etiam, si r sit æqualis 4, erit

$$\bar{x}^{-(4)} = 4 \bar{x}^{\frac{1}{1} \frac{-1}{1}} - \left\{ \begin{array}{c} 4 \bar{x}^{\frac{2}{2} \frac{-2}{2}} \\ 8 \bar{x}^{\frac{1}{1} \frac{-1}{1}} \end{array} \right\} \bar{x} + \left\{ \begin{array}{c} 4 \bar{x}^{\frac{3}{3} \frac{-3}{3}} \\ 24 \bar{x}^{\frac{2}{2} \frac{-1}{1}} \\ 8 \bar{x}^{\frac{1}{1} \frac{-3}{3}} \end{array} \right\} \bar{x} - \left\{ \begin{array}{c} 4 \bar{x}^{\frac{4}{4} \frac{-4}{4}} \\ 32 \bar{x}^{\frac{3}{3} \frac{-2}{2}} \\ 24 \bar{x}^{\frac{2}{2} \frac{-2}{2}} \\ 48 \bar{x}^{\frac{2}{2} \frac{-1}{1}} \end{array} \right\} \bar{x} + \&c.$$

ut in *Proposit. III. Exempl. 5.* Ex quibus evidens est, ubi r est numerus integer unitatem superans, semper in columna numero r & sequentibus aliquos terminos excidere.

E X E M P L U M . II.

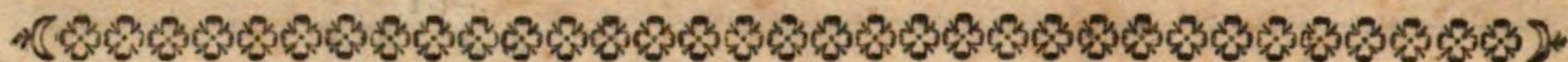
QUod si ponatur $r=1$, tum omnes termini remanebunt, (idque quoniam in tota universali serie non datur multiplicator $r-1$,) & signa quandoque mutabuntur hoc pacto:

$x =$

$$\begin{aligned}
 x = & \frac{1}{a^{(1)}} x^1 - \frac{1}{a^{(2)}} x^2 + \frac{1}{a^{(3)}} x^3 - \frac{1}{a^{(4)}} x^4 + \frac{1}{a^{(5)}} x^5 - \frac{1}{a^{(6)}} x^6 + \frac{1}{a^{(7)}} x^7 - \frac{1}{a^{(8)}} x^8 + \frac{1}{a^{(9)}} x^9 - \frac{1}{a^{(10)}} x^{10} + \dots \\
 & + \frac{1}{a^{(2)}} x^2 - \frac{1}{a^{(3)}} x^3 + \frac{1}{a^{(4)}} x^4 - \frac{1}{a^{(5)}} x^5 + \frac{1}{a^{(6)}} x^6 - \frac{1}{a^{(7)}} x^7 + \frac{1}{a^{(8)}} x^8 - \frac{1}{a^{(9)}} x^9 + \frac{1}{a^{(10)}} x^{10} - \dots \\
 & + \frac{1}{a^{(3)}} x^3 - \frac{1}{a^{(4)}} x^4 + \frac{1}{a^{(5)}} x^5 - \frac{1}{a^{(6)}} x^6 + \frac{1}{a^{(7)}} x^7 - \frac{1}{a^{(8)}} x^8 + \frac{1}{a^{(9)}} x^9 - \frac{1}{a^{(10)}} x^{10} + \dots \\
 & + \frac{1}{a^{(4)}} x^4 - \frac{1}{a^{(5)}} x^5 + \frac{1}{a^{(6)}} x^6 - \frac{1}{a^{(7)}} x^7 + \frac{1}{a^{(8)}} x^8 - \frac{1}{a^{(9)}} x^9 + \frac{1}{a^{(10)}} x^{10} - \dots \\
 & + \frac{1}{a^{(5)}} x^5 - \frac{1}{a^{(6)}} x^6 + \frac{1}{a^{(7)}} x^7 - \frac{1}{a^{(8)}} x^8 + \frac{1}{a^{(9)}} x^9 - \frac{1}{a^{(10)}} x^{10} + \dots \\
 & + \frac{1}{a^{(6)}} x^6 - \frac{1}{a^{(7)}} x^7 + \frac{1}{a^{(8)}} x^8 - \frac{1}{a^{(9)}} x^9 + \frac{1}{a^{(10)}} x^{10} - \dots \\
 & + \frac{1}{a^{(7)}} x^7 - \frac{1}{a^{(8)}} x^8 + \frac{1}{a^{(9)}} x^9 - \frac{1}{a^{(10)}} x^{10} + \dots \\
 & + \frac{1}{a^{(8)}} x^8 - \frac{1}{a^{(9)}} x^9 + \frac{1}{a^{(10)}} x^{10} - \dots \\
 & + \frac{1}{a^{(9)}} x^9 - \frac{1}{a^{(10)}} x^{10} + \dots \\
 & + \frac{1}{a^{(10)}} x^{10} - \dots
 \end{aligned}$$

Verum, si fiat $r=0$, consequens erit, x five $1, = 0 \times flux. ser.$ hoc est, unitatem non posse adæquari seriei fluxionali, quoniam est hæc quantitas constans; neque enim inde concludi debebit, unitatem nihilo æqualem esse; sic etenim ex veritate absurditas deduceretur, quod fieri nequit: At si r sit numerus integer privativus, vel etiam fractus five adfirmativus, five negativus, tum itidem omnes termini remanebunt, & signa varie mutabuntur.

Ufus autem hujus Propositionis hic est, quod posita universalis hac serie $= Q$, si quis ipsius $r x^{\frac{1}{a^{(r-1)}}} x^{\frac{1}{a^{(a+1)}}}$ fluentem primam quæreret juxta *Proposit. V.* eam inveniret congruenter primæ seriei $r x^{\frac{1}{a^{(r)}}} - \overline{r-1} \times Q$, & juxta secundam & reliquas omnes statim Q ; unde adparet, $r x^{\frac{1}{a^{(r)}}} - \overline{r-1} \times Q = Q$ esse, five $r x^{\frac{1}{a^{(r)}}} = r Q$, adeoque $x^{\frac{1}{a^{(r)}}} = Q$, ut in hac Propositione vidimus; quare & fluens prima ipsius $r x^{\frac{1}{a^{(r-1)}}} x^{\frac{1}{a^{(a+1)}}}$ erit $r x^{\frac{1}{a^{(r)}}} - \overline{r-1} \times x^{\frac{1}{a^{(r)}}}$, hoc est $x^{\frac{1}{a^{(r)}}}$, id quod non nisi ope hujus Propositionis detegi potuisset.



A D D I T A M E N T U M.

U Num est, mi Lector, quod ad illustrationem eorum, quæ in hocce Opusculo tradidimus, adjicere necessarium duxi; me videlicet tam quod fluxiones quam fluentes quantitates illas, quæ respectu reliquorum terminorum fluxionalium infinite parvæ vel magnæ erant, neglexisse: Quod ut exemplo manifestum fiat, consideretur tantummodo fluxio prima ipsius

$\dot{R} \dot{S}$, quæ proprie quidem est $\dot{R} \dot{S}^{a+b+1} + \dot{R}^{a+1} \dot{S}^b + \dot{R}^{a+1} \dot{S}^{b+1}$; verum

quoniam $\dot{R} \dot{S}^{a+1 b+1}$ est infinite exigua, si comparetur cum $\dot{R} \dot{S}^{a b+1}$ &

$\dot{R} \dot{S}^{a+1 b}$, hinc prædictæ quantitatis nullam prorsus rationem habui; & sic deinceps quoad cæteras fluxiones cujusvis ordinis tam adfirmativas quam negativas, qua in re omnium Eruditorum, quorum scripta super hoc argumento in manus pervenire meas, exemplum sequutus fui: Cæterum, quoniam post typis descriptam jam magnam Opellæ nostræ partem ad aures meas delatum fuit, Virum longe Doctissimum Colinum Mac Laurin in Academia Edinburgensi Matheseos Professore Inclytum, qui edito Geometriæ Organicæ Opere, quo nihil hactenus visum in eo genere sublimius, Erudito Orbi dudum innotuit, identidem super Fluxionum materia admirandum Opus prælo commisisse, in quo, ut ego quidem accepi, quantitates illas infinite exiguas minime negligi patitur, operæ pretium existimavi fore, si juxta Celeberrimi Viri mentem duarum saltem quantitatum variabilium (nam de pluribus jam non ero sollicitus) in semet ductarum fluxiones & fluentes cujus-

libet ordinis ob oculos ponerem: Igitur ipsius $\dot{R} \dot{S}^{a b}$ varietatem generis (m) æqualem dico fore alterutri ex duabus sequentibus seriebus.

$\dot{S}^{b+m}$
 $\dot{S}$

$$\begin{aligned}
 & \dot{S} \dot{R} + \frac{m}{1} \left\{ \begin{matrix} \dot{S} \\ b+m \\ \dot{S} \end{matrix} \right\} \dot{R} + \frac{m}{1} \times \frac{m-1}{2} \left\{ \begin{matrix} \dot{S} \\ b+m-1 \\ 2 \dot{S} \\ b+m \\ \dot{S} \end{matrix} \right\} \dot{R} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \left\{ \begin{matrix} \dot{S} \\ b+m-2 \\ 3 \dot{S} \\ b+m-1 \\ 3 \dot{S} \\ b+m \\ \dot{S} \end{matrix} \right\} \dot{R} \\
 & + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \left\{ \begin{matrix} \dot{S} \\ b+m-3 \\ 4 \dot{S} \\ b+m-2 \\ 6 \dot{S} \\ b+m-1 \\ 4 \dot{S} \\ b+m \\ \dot{S} \end{matrix} \right\} \dot{R} + \&c.
 \end{aligned}$$

Item

$$\begin{aligned}
 & \dot{R} \dot{S} + \frac{m}{1} \left\{ \begin{matrix} \dot{R} \\ a+m \\ \dot{R} \end{matrix} \right\} \dot{S} + \frac{m}{1} \times \frac{m-1}{2} \left\{ \begin{matrix} \dot{R} \\ a+m-1 \\ 2 \dot{R} \\ a+m \\ \dot{R} \end{matrix} \right\} \dot{S} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \left\{ \begin{matrix} \dot{R} \\ a+m-2 \\ 3 \dot{R} \\ a+m-1 \\ 3 \dot{R} \\ a+m \\ \dot{R} \end{matrix} \right\} \dot{S} \\
 & + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \left\{ \begin{matrix} \dot{R} \\ a+m-3 \\ 4 \dot{R} \\ a+m-2 \\ 6 \dot{R} \\ a+m-1 \\ 4 \dot{R} \\ a+m \\ \dot{R} \end{matrix} \right\} \dot{S} + \&c.
 \end{aligned}$$

Eadem etiam ratione ipsius $\dot{x}$ fluxio seu decrementum (hæc enim

enim mihi synonyma sunt) primi ordinis tum non erit
 $\frac{r^{(r-1)}}{a^{(a+1)}} x$; sed utique $\frac{r^{(r-1)}}{a^{(a+1)}} x + \frac{r^{(r-2)}}{a^{(a+1)}} x + \frac{r^{(r-3)}}{a^{(a+1)}} x$

+ &c. quæ series æqualis est $x + x - x$. Pluribus hæc omnia exsequi & clarius exponere vellem, nisi Typographus, cui pagellæ hæ jam jam tradendæ erunt, negotio huic sufflamen injiceret: Itaque, quod nos temporis angustiis exclusi absolvere jam non possumus, aliis felicioris ingenii ulterius exsequendum, quodque rudiori penicillo hic delineavimus, poliendum relinquimus.

F I N I S.



MISCELLANEA.

K

MISCELLANEA.

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MISCELLANEORUM

C A P U T I.

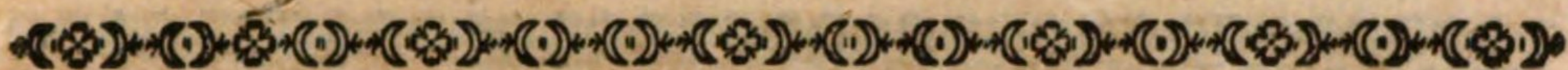
Quo exhibetur quadratura lineæ curvæ, cujus natura exprimitur
per hanc æquationem, $y = x \times x + a^{\frac{1}{m}}$.



Otum est, curvæ lineæ quadraturam exprimi per
fluentem primam ipsius y x ; id quod si huic
exemplo adaptare velimus, capienda erit fluens
quantitatis $x \times x + a^{\frac{1}{m}}$ x : Fiet autem illud se-
quenti modo quam brevissime. Ponatur $x + a = w$,
adeoque $x = w - a$, & $x^{\frac{1}{m}} = w^{\frac{1}{m}}$; unde fluens adsumenda erit ip-
sius $w - a \times w^{\frac{1}{m}}$ w , sive $w^{\frac{m+1}{m}}$ $w - a w^{\frac{1}{m}}$ w , quæ proinde æqua-
lis erit $\frac{m}{2m+1} w^{\frac{2m+1}{m}} - \frac{am}{m+1} w^{\frac{m+1}{m}}$, sive $\frac{m}{m+1 \times 2m+1} \times mw + w - 2am - a$
 $\times w^{\frac{m+1}{m}}$, & substituendo valorem ipsius x , habebimus, $\frac{m}{m+1 \times 2m+1}$
 $\times mx + x - am \times x + a^{\frac{m+1}{m}}$: Deinde, quoniam origo ipsius x
incognita est, in prædicta fluente pro x substituendum est 0,
sicque habebimus $-\frac{m^2}{m+1 \times 2m+1} \times a^{\frac{2m+1}{m}}$, quod si ab eadem fluente
subducatur, erit quæsitæ curvæ quadratura æqualis $\frac{m}{m+1 \times 2m+1} \times$
 $mx + x - am \times x + a^{\frac{m+1}{m}} + \frac{m^2}{m+1 \times 2m+1} a^{\frac{2m+1}{m}}$; id quod etiam ad pa-
rabolam cujuscunque ordinis, quæ (posito parametro = 1,)
hac æquatione $y = x^{\frac{p}{q}}$ exprimitur, adplicari potest; quandoqui-
dem;

dem, si æquatio illa $y = x \times x^{-\frac{p-q}{q}}$ conferatur cum altera $y = x \times \frac{1}{x+a}^{\frac{1}{m}}$, erit $a=0$, $m=\frac{q}{p-q}$, adeoque parabolæ cujusvis quadratura erit $\frac{q}{p-q} x^{-\frac{p-q}{q}}$, quod cum veritate consentaneum est. Ut adeo ex hisce appareat, lapsum fuisse Cl. Stonium, quum in *Analysi Infinitorum Sect. III. Exempl. 6. pag. m. 44. in Edit. Paris. 1735.* quæsitam prædictæ curvæ quadraturam scripsit esse $\frac{m}{m+1} \times x + a^{\frac{m+1}{m}} - \frac{m}{m+1} \times a^{\frac{m+1}{m}}$, quod utique verum foret, si y data fuisset æqualis $x + a^{\frac{1}{m}}$; quum tamen in hoc casu sit $y = x \times \frac{1}{x+a}^{\frac{1}{m}}$: Id quod tamen non ideo hic commemoravimus, ut diligentissimum deque hac Matheseos parte optime meritum Virum vel aliqua ratione debita laude defraudaremus; (quippe quem errorem calami festinantia tribuere licet, qualesque si & in hoc Opusculo forte conspiciantur, ut eos æqui bonique consulere velit Lector, eum inprimis rogatum volumus,) verum, ne Tyrones auctoritate Clarissimi Viri nimium confisi in errorem seducantur; quum sæpius deprehenderim, Eruditissimos Viros, dum aliorum scrinia compilant, ne in manifestissimis quidem calculi erroribus ab iisdem recedere.





C A P U T I I.

Quo exhibetur subtangens lineæ Ellipticæ cujuscunque ordinis.

Constat unicuique, subtangentem curvæ lineæ exprimi per $y^{\frac{1}{r-1}} x^{\frac{1}{r-1}}$: Proprietas vero lineæ Ellipticæ cujuslibet ordinis exprimitur per hanc æquationem $\frac{a^m}{b^p} y^n = \overline{a^q - x^q}^{(r)} \times \overline{x}^{(m+n-p-qr)}$; adeoque erit $y^{\frac{1}{r-1}} x^{\frac{1}{r-1}}$ æqualis $a^{-\frac{m}{n}} b^{\frac{p}{n}} \times \overline{a^q - x^q}^{(\frac{r}{n})} \times \overline{x}^{(\frac{m+n-p-qr}{n})}$, & $y^{\frac{1}{r-1}} = a^{-\frac{m}{n}} b^{\frac{p}{n}} \times \frac{m+n-p-qr}{n} \times \overline{a^q - x^q}^{(\frac{r}{n})} \times \overline{x}^{(\frac{m+n-p-qr}{n})} x - a^{-\frac{m}{n}} b^{\frac{p}{n}} \times \frac{qr}{n} \times \overline{a^q - x^q}^{(\frac{r-n}{n})} \times \overline{x}^{(\frac{m+qn-p-qr}{n})}$, per quem diviso $y^{\frac{1}{r-1}} x^{\frac{1}{r-1}}$, erit $\frac{y^{\frac{1}{r-1}} x^{\frac{1}{r-1}}}{y^{\frac{1}{r-1}} x^{\frac{1}{r-1}}} = \frac{nx \times a^q - x^q}{m+n-p-qr \times a^q - x^q - qr x^q} = \frac{nx \times a^q - x^q}{m+n-p-qr \times a^q - m+n-p \times x^q}$.

Si quis jam desideret subtangentem Ellipseos primi generis, proprietas ejus indicabitur hac æquatione $\frac{a^2}{b^2} y^2 = \overline{a - x} \times x$; unde erunt $m=2$, $n=2$, $p=2$, $q=r=1$, ideoque $\frac{y^{\frac{1}{r-1}} x^{\frac{1}{r-1}}}{y^{\frac{1}{r-1}} x^{\frac{1}{r-1}}} = \frac{2x \times a - x^2}{a - 2x} = \frac{a x - x^2}{a - x}$.





C A P U T I I I.

Quo inquiritur in radicem æquationis hujus, $\bar{x}^{(pm)} + \frac{m}{1} \bar{x}^{(p \times m - 1)} \bar{a}^{(q)}$
 $+ \frac{m}{1} \times \frac{m-1}{2} \bar{x}^{(p \times m - 2)} \bar{a}^{(2q)} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \bar{x}^{(p \times m - 3)} \bar{a}^{(3q)} + \dots - \bar{a}^{(mq)} = \bar{b}^{(r)}$

Dico radicem hujus æquationis fore hanc: $x = \sqrt[p]{-\bar{a}^{(q)} + \bar{a}^{(mq)} + \bar{b}^{(r)}}$, per quam in dato casu singulari

divisa æquatione reliquæ radices patefient: Ejus autem rei demonstratio admodum facilis est; quum data fuerit æquatio

$x^p + \bar{a}^{(q)} - \bar{a}^{(mq)} = \bar{b}^{(r)}$, adeoque $x^p + \bar{a}^{(q)} = \bar{a}^{(mq)} + \bar{b}^{(r)}$, unde

$x = \sqrt[p]{-\bar{a}^{(q)} + \bar{a}^{(mq)} + \bar{b}^{(r)}}$. Itaque, si ponatur exempli causa

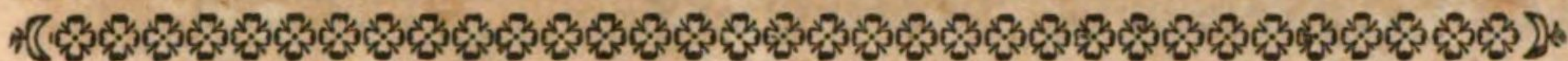
$m=1$, erit $x^p = \bar{b}^{(r)}$, & proinde $x = \sqrt[p]{\bar{b}^{(r)}}$. Posito $m=2$, prodibit æquatio, $x^{2p} + 2x^p \bar{a}^{(q)} = \bar{b}^{(r)}$, sive $x^{2p} = -2\bar{a}^{(q)}x^p + \bar{b}^{(r)}$, unde

quantitas x æqualis fit $\sqrt[p]{-\bar{a}^{(q)} + \sqrt{4\bar{a}^{(2q)} + \bar{b}^{(r)}}}$, & sic vides, ex hac serie ordinariam æquationis quadraticæ resolutionem facili negotio deduci posse.

Quod si fiat $m=3$, erit æquatio, $x^{3p} + 3x^{2p} \bar{a}^{(q)} + 3x^p \bar{a}^{(2q)} = \bar{b}^{(r)}$, seu $x^{3p} = -3\bar{a}^{(q)}x^{2p} - 3\bar{a}^{(2q)}x^p + \bar{b}^{(r)}$; unde $x =$

$\sqrt[p]{-\bar{a}^{(q)} + \sqrt[3]{\bar{a}^{(3q)} + \bar{b}^{(r)}}}$; & sic de reliquis: Et poterit etiam pro m

adhiberi non tantum numerus affirmativus, sed etiam negativus; imo etiam fractus, sive is nihilo major sit, sive minor; ut proinde æquatio hæc, cujus resolutionem jam exhibui, certo respectu dici possit universalis.



C A P U T I V.

*Continens Problematum quorundam difficiliorum resolutionem
Analyticam.*

O Ccasio in novam quorundam Problematum resolutionem inquirendi nata fuit ex eo, quod Abrahamus de Graaf in sua *ad Algebram Introductione*, quæ vernacula lingua Amstelodami A°. 1706. lucem adspexit, problemata nonnulla, in quibus duæ quantitates incognitæ eodem modo inveniuntur, adseveraverit, solvi haud posse per x & y , nisi incidamus in æquationem quadratica majorem; quam rem cum semper antehac miratus fuisset, de ejus veritate tandem dubitare coepi, ipsaque problemata ope æquationis duarum dimensionum per x & y resolvi; qua ratione etiam Cl. Ozanamum in *Novis Algebræ Elementis* ibidem loci A°. 1702. Gallica lingua editis *Lib. III. Sect. 3. pag. 323. & seq.* quæstionem quandam, quam Franciscus a Schooten in *Comment. ad Lib. I. Geometr. Ren. des Cartes pag. m. 158.* per $x + y$ & $x - y$ resolverat, simpliciter per x & y enodasse postmodum deprehendi; at quoniam Cl. Ozanamus in quæstione illa (meo judicio sic satis facili) explicanda calculo valde prolixo & tædioso utitur, ne quis existimaret Fr. Schotenii & Abr. de Graaf methodo citius ac nostra quæsitæ quantitates obtineri, non peccaturum me fore existimavi, si & illius Problematis compendiosam admodum solutionem reliquis adnecterem.

QUÆS.

Q U Æ S T I O I.

Investigare duos numeros, quorum productum æquale est a, & quadratum eorum differentiae æquale summæ numerorum. De Graaf Quæst. 145.

SI pro uno numero adhibeatur x , & pro altero y , prodibunt duæ æquationes, prior $xy = a$, & altera $x^2 - 2xy + y^2 = x + y$, cui posteriori æquationi si addamus $4xy = 4a$, erit $x^2 + 2xy + y^2 = x + y + 4a$, quare $x + y = \frac{1}{2} \pm \sqrt{\frac{1}{4} + 4a}$, & proinde juxta secundam æquationem datam; erit $x - y = \sqrt{\frac{1}{2} \pm \sqrt{\frac{1}{4} + 4a}}$, unde $x = \frac{1}{4} \pm \sqrt{\frac{1}{16} + a} + \sqrt{\frac{1}{8} \pm \frac{1}{2} \sqrt{\frac{1}{16} + a}}$, ac $y = \frac{1}{4} \pm \sqrt{\frac{1}{16} + a} - \sqrt{\frac{1}{8} \pm \frac{1}{2} \sqrt{\frac{1}{16} + a}}$. Vel brevius, ponendo $\frac{1}{2} \pm \sqrt{\frac{1}{4} + 4a}$ (quod æquale erat $x + y$, & $\overline{x - y^2}$), $= p^2$, erit $x = \frac{p+1 \times p}{2}$, & $y = \frac{p-1 \times p}{2}$. Posito $a = 60$, erit $p = 4$, adeoque $x = 10$, & $y = 6$; nam alter valor ipsius p adeoque etiam quantitatum x & y est imaginarius.

Q U Æ S T I O II.

Invenire duos numeros, quorum productum æquale est eorum summæ, & quorum summa quadratorum æqualis est a. De Graaf Quæst. 147.

SI x statuatur unus, & y alter numerus, erit $xy = x + y$, & $x^2 + y^2 = a$, cui posteriori æquationi si addatur $2xy = 2x + 2y$, erit $x^2 + 2xy + y^2 = 2x + 2y + a$; unde $x + y = 1 \pm \sqrt{1 + a}$. Jam cognita est summa & productum duorum numerorum; quare

quare juxta vulgarem operandi rationem invenitur $x = \frac{1 + \sqrt{a+1} + \sqrt{a-2+2\sqrt{a+1}}}{2}$ & $y = \frac{1 + \sqrt{a+1} - \sqrt{a-2+2\sqrt{a+1}}}{2}$, vel ponendo $1 + \sqrt{1+a} = p$, erit $x = \frac{p + \sqrt{p^2-4p}}{2}$, & $y = \frac{p - \sqrt{p^2-4p}}{2}$; vel adhuc brevius, si pro $1 + \sqrt{1+a}$ ponamus $\frac{q}{q-1}$, erit $x = q$, & $y = \frac{q}{q-1}$. Posito $a=80$, erit $p=10$, unde $x=q=5 + \sqrt{15}$, & $y=5 - \sqrt{15}$; aut etiam $p=-8$, sicque $x=-4 + 2\sqrt{6}$, ac $y=-4 - 2\sqrt{6}$.

Q U Æ S T I O I I I.

Duos repperire numeros, ita ut eorum differentia multiplicata cum quadrato summae equalis sit a, & differentia ducta in quadratum alterutrius equalis sit b. Idem Quæst. 148.

POsito uno numero x & altero y , prodibunt hæ æquationes, $x - y \times x + y^2 = a$, & $x - y \times y^2 = b$. Igitur divisa priori per posteriorem, erit $\frac{x+y}{y^2} = \frac{a}{b}$, ideoque $\frac{x+y}{y} = \sqrt{\frac{a}{b}}$, & $x = -1 + \sqrt{\frac{a}{b}} \times y$, unde juxta secundam æquationem erit $y^3 \times -2 + \sqrt{\frac{a}{b}} = b$, adeoque $y = \sqrt[3]{\frac{b}{-2 + \sqrt{\frac{a}{b}}}}$, & $x = -1 + \sqrt{\frac{a}{b}} \times \sqrt[3]{\frac{b}{-2 + \sqrt{\frac{a}{b}}}}$. Abr. de Graaf pro ea quantitate, quæ hic litera y designata fuit, habet $\sqrt[3]{\frac{2bb+b\sqrt{ab}}{a-4b}}$: Sed quis non videt, & numeratorem & denominatorem hujus fractionis dividi posse per $2b + \sqrt{ab}$, & sic ipsam y eo, quo dixi, modo inveniri? Posito $a=363$, & $b=48$, erit $\sqrt{\frac{a}{b}} = \frac{11}{4}$, unde $y=4$, & $x = \frac{7}{4} \times 4 = 7$. Sic vides simplicem prodire æquationem, cum methodo Abrⁱ de Graaf incideres in æquationem duarum dimensionum.

Q U Æ S T I O I V.

Invenire duos numeros, quorum productum additum eorum summae æquale est a, & aggregatum numerorum ablatum a summa quadratorum æquale ipsi b. Idem Quæst. 149.

ÆQuationes jam prodeunt hæ, $xy + x + y = a$, & $x^2 + y^2 - x - y = b$. Huic secundæ æquationi si addamus $2xy + 2x + 2y = 2a$, habebimus $x^2 + 2xy + y^2 + x + y = 2a + b$, five $x + y^2 = -x - y + 2a + b$; unde $x + y$ æquabitur $-\frac{1}{2} \pm \sqrt{\frac{1}{4} + 2a + b} = p$. Jam igitur $xy = a - p$, unde $x = \frac{p + \sqrt{p^2 + 4p - 4a}}{2}$, & $y = \frac{p - \sqrt{p^2 + 4p - 4a}}{2}$, vel brevius adhuc, si pro $-\frac{1}{2} \pm \sqrt{\frac{1}{4} + 2a + b}$ ponamus $\frac{q^2 + a}{q + 1}$, erit $x = q$, & $y = \frac{a - q}{q + 1}$. Posito $a = 143$, $b = 266$, erit $p = 23$, & $x = q = 15$, ac $y = 8$; vel etiam $p = -24$, sed tunc valor quantitatum x & y est imaginarius.

Q U Æ S T I O V.

Duos investigare numeros, quorum differentia æqualis est a, & differentia cuborum ducta in summam quadratorum æqualis b. Idem Quæst. 176.

PRior æquatio est, $x - y = a$, altera $x^3 - y^3 \times x^2 + y^2 = b$, qua divisa per priorem erit $x^2 + xy + y^2 \times x^2 + y^2 = \frac{b}{a}$, five $x^2 - 2xy + y^2 + 3xy \times x^2 - 2xy + y^2 + 2xy = \frac{b}{a}$, hoc est, $a^2 + 3xy \times a^2 + 2xy = \frac{b}{a}$: Proinde $6x^2y^2 + 5a^2xy + a^4 = \frac{b}{a}$, adeoque $x^2y^2 = -\frac{5}{6}a^2xy + \frac{b - a^5}{6a}$; unde $xy = -\frac{5}{12}a^2 \pm \frac{1}{12}\sqrt{\frac{a^5 + 24b}{a}} = p$, & quoniam jam differentia numerorum est nota, erit $x = \frac{a + \sqrt{a^2 + 4p}}{2}$, & $y = \frac{-a + \sqrt{a^2 + 4p}}{2}$; vel brevius, si pro p scribamus $aq + q^2$, erit $x = a + q$, & $y = q$. Posito $a = 1$, $b = 247$, erit $p = 6$, ac $y = q = 2$,

$q=2$, & $x=3$. Apparet jam falsissimum esse, quod Abr. de Graaf ait, quæstionem hanc aliter solvi non posse, nisi pro uno numero adhibeatur $x-\frac{1}{2}a$, & pro altero $x+\frac{1}{2}a$, quin incidamus in æquationem quatuor dimensionum.

Q U Æ S T I O V I.

Datum numerum a in duas partes dividere, ita ut summa cuborum ducta in summam quadratorum æqualis sit ipsi b. Idem Quæst. 209.

POnatur pro una parte x & pro altera y , erunt æquationes, $x+y=a$, & $x^3+y^3 \times x^2+y^2=b$. Posteriori per priorem divisa erit $x^2-xy+y^2 \times x^2+y^2=\frac{b}{a}$, sive $x^2+2xy+y^2-3xy \times x^2+2xy+y^2-2xy=\frac{b}{a}$, hoc est $a^2-3xy \times a^2-2xy=\frac{b}{a}$. Proinde $6x^2y^2-5a^2xy+a^4=\frac{b}{a}$, adeoque $x^2y^2=\frac{5}{6}a^2xy+\frac{b-a^4}{6a}$, sicque $xy=\frac{5}{12}a^2 \pm \frac{1}{12}\sqrt{\frac{a^4+24b}{a}}=p$. Et, quoniam jam summa est nota, erit $x=\frac{a+\sqrt{a^2-4p}}{2}$, & $y=\frac{a-\sqrt{a^2-4p}}{2}$, vel si pro p substituamus $aq-q^2$, erit $x=a-q$, & $y=q$. Sic ut quæstio hæc tantummodo signorum diversitate a priori variet. Posito $a=5$, & $b=455$, erit $p=6$, sicque $y=q=2$, & $x=3$; altera enim radix quantitatum x & y est imaginaria.

Q U Æ S T I O V I I.

Invenire duos numeros, quorum summa æqualis est eorundem producto, & quorum summa quadratorum addita summæ numerorum æqualis est a. Idem Quæst. 227.

Æquationes in hac specie dantur hæ, $xy=x+y$, & $x^2+y^2+x+y=a$. Huic posteriori si addas $2xy-2x-2y$
L 2 =0,

$=0$, erit $x^2 + 2xy + y^2 - x - y = a$; unde $x + y = xy = \frac{1}{2} \pm \sqrt{\frac{1}{4} + a}$
 $=p$; adeoque si ab $x^2 + xy + y^2 = a$ subducamus $3xy = 3p$, &
 ex residuo radicem extrahamus, erit $x - y = \sqrt{a - 3p}$, & proin-
 de $x = \frac{p + \sqrt{a - 3p}}{2}$, & $y = \frac{p - \sqrt{a - 3p}}{2}$. Posito $a = 12$, erit $p = 4$, ideo-
 que $x = 2$, & $y = 2$. Alii numeri integri adhiberi non possunt,
 qui prædictam conditionem, ut videl. eorum summa producto
 æqualis sit, admittunt; sed tantummodo vel fracti vel irratio-
 nales: Quod vero ad alterum valorem adinet, erit $p = -3$,
 hincque $x = -\frac{3}{2} + \frac{1}{2}\sqrt{21}$, & $y = -\frac{3}{2} - \frac{1}{2}\sqrt{21}$.

Q U Æ S T I O V I I I.

*Duos numeros reperire, quorum summa, productum, & cuborum
 summa inter se æqualia sunt. Idem Quæst. 277.*

Sit x unus, & y alter numerus, erit $x + y = xy$, item
 $x^3 + y^3 = x + y$. Posteriori æquatione divisa per $x + y$, erit
 $x^2 - xy + y^2 = 1$. Huic novæ æquationi si addatur $3xy = 3x$
 $+ 3y$, erit $x^2 + 2xy + y^2 = 3x + 3y + 1$; unde $x + y = \frac{3 \pm \sqrt{13}}{2} = xy$.
 Jam si ab $x^2 - xy + y^2 = 1$, subducamus $xy = \frac{3 \pm \sqrt{13}}{2}$, habebimus
 $x - y = \frac{-1 \pm \sqrt{13}}{2}$, adeoque $x - y = \sqrt{-\frac{1}{2} \pm \frac{1}{2}\sqrt{13}}$. Nunc, quoniam
 cognita est summa & differentia quantitatum x & y , facillime
 invenitur $x = \frac{3 \pm \sqrt{13} + 2\sqrt{-\frac{1}{2} \pm \frac{1}{2}\sqrt{13}}}{4}$, & $y = \frac{3 \pm \sqrt{13} - 2\sqrt{-\frac{1}{2} \pm \frac{1}{2}\sqrt{13}}}{4}$.

Q U Æ S T I O I X.

*Invenire duos numeros, quorum differentia, productum, item
 cuborum differentia æqualia sunt. Idem Quæst. 278.*

Æquationes jam prodeunt hæ, $x - y = xy$, & $x^3 - y^3 = x - y$.
 Posteriori æquatione divisa per $x - y$, erit $x^2 + xy + y^2 = 1$,
 unde

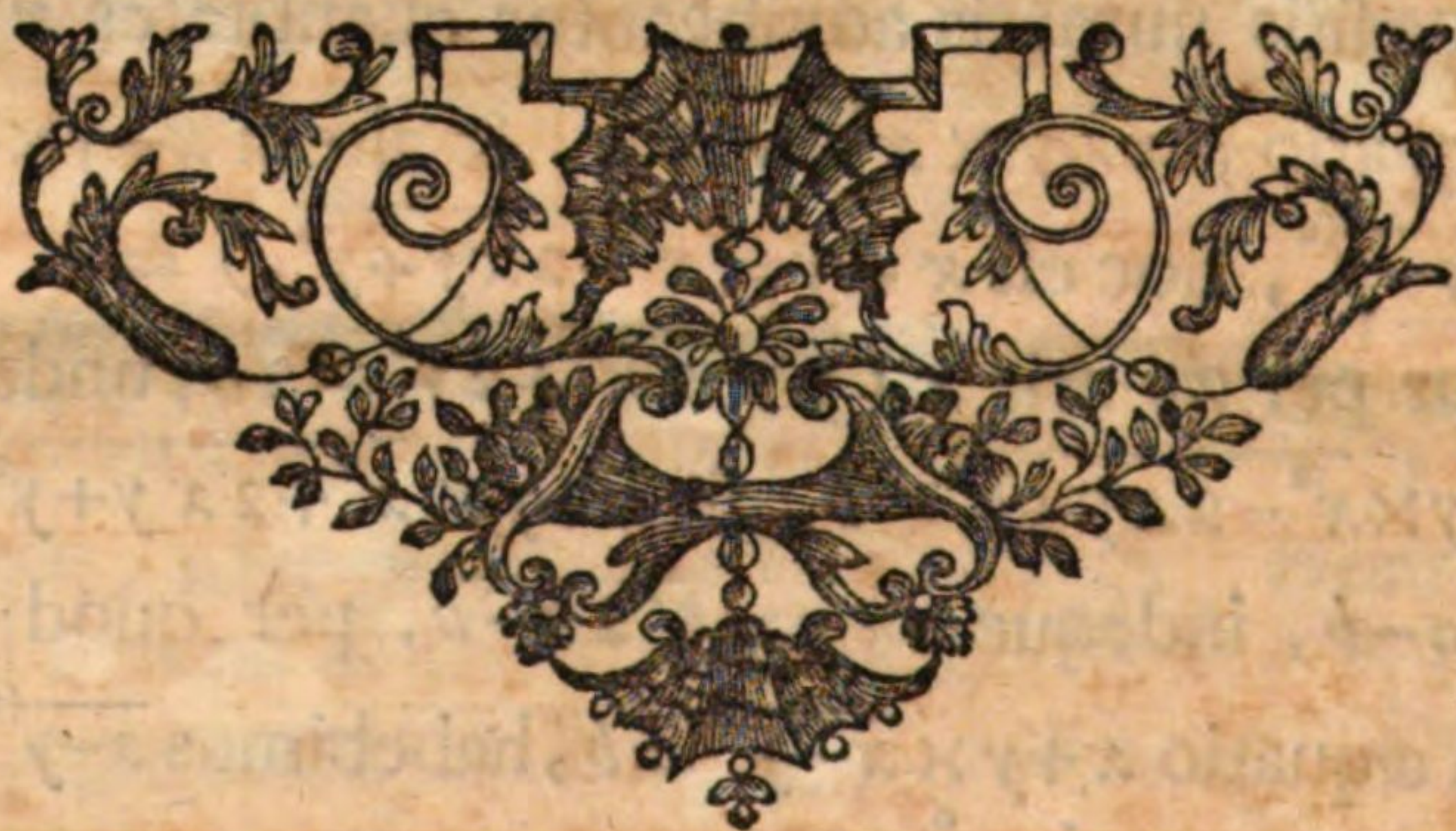
unde si subducatur $3xy = 3x - 3y$, erit $\overline{x-y}^{(2)} = -3x + 3y + 1$, unde $x - y = \frac{-3 \pm \sqrt{13}}{2} = xy$. Porro, si prædictæ æquationi, $x^2 + xy + y^2 = 1$, addamus $xy = \frac{-3 \pm \sqrt{13}}{2}$, habebimus $\overline{x+y}^{(2)} = \frac{-1 \pm \sqrt{13}}{2}$, adeoque $x + y = \sqrt{-\frac{1}{2} \pm \frac{1}{2} \sqrt{13}}$. Jam, quoniam etiam $x - y$ modo innotuit, erit $x = \frac{-3 \pm \sqrt{13} + 2\sqrt{-\frac{1}{2} \pm \frac{1}{2} \sqrt{13}}}{4}$, & $y = \frac{+3 \mp \sqrt{13} + 2\sqrt{-\frac{1}{2} \pm \frac{1}{2} \sqrt{13}}}{4}$; ut adeo hæc quæstio a priori tantum signis differat.

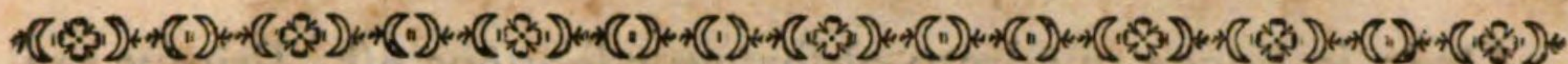
Q U Æ S T I O X.

Duos invenire numeros, quorum summa in quadratorum summam ducta æqualis est a, & quorum differentia multiplicata per differentiam quadratorum æquatur ipsi b. Ozanam
Lib. III. Sect. 3. Probl. 1.

SI pro quæsitis numeris adhibeantur quantitates x & y , erit una æquatio, $x + y \times \overline{x^2 + y^2} = a$, & altera $x - y \times \overline{x^2 - y^2}$ (si-
ve $x + y \times \overline{x - y}^{(2)}$, hoc est $x + y \times \overline{x^2 - 2xy + y^2}$,) $= b$. Si prior multiplicetur per 2, erit $x + y \times \overline{2x^2 + 2y^2} = 2a$, unde subducta altera, $x + y \times \overline{x^2 - 2xy + y^2} = b$, fit $x + y \times \overline{x^2 + 2xy + y^2}$ (hoc est $\overline{x+y}^{(3)}$) $= 2a - b$, indeque $x + y = \sqrt[3]{2a - b}$, per quod si dividatur secunda æquatio $x + y \times \overline{x - y}^{(2)} = b$, habebimus $\overline{x - y}^{(2)} = \frac{b}{\sqrt[3]{2a - b}} = \sqrt[6]{\frac{b^3}{2a - b}}$, indeque $x - y = \sqrt[6]{\frac{b^3}{2a - b}}$; & hinc jam facile innotescit $x = \frac{1}{2} \sqrt[3]{2a - b} + \frac{1}{2} \sqrt[6]{\frac{b^3}{2a - b}}$, & $y = \frac{1}{2} \sqrt[3]{2a - b} - \frac{1}{2} \sqrt[6]{\frac{b^3}{2a - b}}$. Posito $a = 715$, $b = 99$, erit $x = \frac{11+3}{2} = 7$, ac $y = \frac{11-3}{2} = 4$. Reliquas quæstiones, quas Jac. Ozanamus *dict. loc.* subjungit, & more suo fuscè persequitur, data opera prætereo, quum haud absimilis naturæ sint, ac proinde a Lectore in Analyfi aliquatenus versato juxta nostram methodum haud difficulter resolvi queant.

DAntur porro & plura alia Problemata, quorum solutiones habeo longe elegantiores iis, quas dedit Abr. de Graaf; præcipue, in quibus agitur de Progressionibus Arithmeticis vel Geometricis; sed quoniam hic prima Algebræ elementa non trado, illa omittere satius duxi. Ea vero, quorum solutionem hoc Capite exhibui, vel eam ob causam prætermittere nolui, quod & Magnos Viros hac in parte Cl. de Graaf sequutosprehenderim, eosque inter vel primo nominandum loco τὸν πάνυ, at nuper eheu! immatura morte nobis ereptum 's Gravesandium (a cujus ore in Physicis & Astronomicis per biennium haud ita pridem pendere mihi licuisse plurimum gratulor,) in *Element. Arithmet. universal. pag. 81. & seq.*





C A P U T V.

Quo exhibetur quadrati magici imparis quocunque loculorum dispositio universalis.

NOta est unicuique magici quadrati natura, quod videl. summa terminorum in quibusvis quadrati columnis tam horizontalibus, quam verticalibus, imo & diagonalibus semper sit eadem. Hisce jam positis detur progressio Arithmetica, p , $p+q$, $p+2q$, &c. ejusque omnes termini collocandi sint in quadrato magico, ubi numerus loculorum in quavis columna est n , & proinde per totum quadratum n^2 , erit prædicti quadrati dispositio sequens.

	I	K	L	M	N	O	P
ι	$p + \frac{n^2-7}{2}q$	$p + \frac{n^2-3}{2}q$	$p + \frac{n^2-2n-5}{2}q$	$p + \frac{n^2-n-2}{2}q$	$p + \frac{n^2-4n-3}{2}q$	$p + \frac{n^2-2n-1}{2}q$	$p + \frac{n^2-6n-1}{2}q$
κ	$p + \frac{n^2-3}{2}q$	$p + \frac{n^2-5}{2}q$	$p + \frac{n^2-2}{2}q$	$p + \frac{n^2-2n-3}{2}q$	$p + \frac{n^2-n-1}{2}q$	$p + \frac{n^2-4n-1}{2}q$	$p + \frac{n^2-3n}{2}q$
λ	$p + \frac{n^2+2n-5}{2}q$	$p + \frac{n^2-2}{2}q$	$p + \frac{n^2-3}{2}q$	$p + \frac{n^2-1}{2}q$	$p + \frac{n^2-2n-1}{2}q$	$p + \frac{n^2-2n}{2}q$	$p + \frac{n^2-4n+1}{2}q$
μ	$p + \frac{n^2-2n-2}{2}q$	$p + \frac{n^2+2n-3}{2}q$	$p + \frac{n^2-1}{2}q$	$p + \frac{n^2-1}{2}q$	$p + \frac{n^2-n}{2}q$	$p + \frac{n^2-2n+1}{2}q$	$p + \frac{n^2-2n+1}{2}q$
ν	$p + \frac{n^2+4n-3}{2}q$	$p + \frac{n^2-1}{2}q$	$p + \frac{n^2+2n-1}{2}q$	p	$p + \frac{n^2+1}{2}q$	$p + \frac{n^2-n+1}{2}q$	$p + \frac{n^2-2n+3}{2}q$
ξ	$p + \frac{n^2-1}{2}q$	$p + \frac{n^2+4n-1}{2}q$	$p + nq$	$p + \frac{n^2+2n+1}{2}q$	$p + q$	$p + \frac{n^2+3}{2}q$	$p + \frac{n^2-n+2}{2}q$
θ	$p + \frac{n^2+6n-1}{2}q$	$p + 2nq$	$p + \frac{n^2+4n+1}{2}q$	$p + \frac{n^2+1}{2}q$	$p + \frac{n^2+2n+3}{2}q$	$p + 2q$	$p + \frac{n^2+5}{2}q$

Ordo

Ordo in hocce quadrato observatus sequens est. Primus terminus Arithmeticae progressionis est p occupans loculum ν M; secundus $p+q$ collocatus in cellula ξ N; tertius $p+2q$, cujus situs est in loculo σ O; quartus $p+3q$, si quadratum continuaretur, locum inveniret in cellula π P; & sic deinceps. Ultimus, & e regione primi positus terminus est $p+\overline{n^2-1}.q$, occupans loculum λ M: Medius vero est $p+\frac{n^2-1}{2}q$, positus in medio totius quadrati, scil. cellula μ M, & ab eo initium sumi oportet, si rationem progressionis in hoc quadrato observatam recte percipere velimus: Igitur, qui in linea transversa ad ejus dextram partem collocati sunt termini, continuo quantitate q adcrefcunt; qui vero ad sinistram, eodem modo decrefcunt: Sic in cellula ν N habes $p+\frac{n^2+1}{2}q$; in loculo ξ O repperis $p+\frac{n^2+3}{2}q$, & ita porro: E contra in cellula λ L videre est $p+\frac{n^2-3}{2}q$; in loculo κ K cernis $p+\frac{n^2-5}{2}q$, & sic deinceps; ut adeo omnes termini hujus columnae transversae sint in progressionem Arithmetica, cujus excessus est q . Deinde si ab hac linea obliqua ad inferiorem descendamus, erit illius medius terminus $p+\frac{n^2+2n-1}{2}q$, cujus situs est in loculo ν L: Qui ad dextram ejus collocati sunt, itidem quantitate q adcrefcunt, qui vero ad sinistram, decrefcunt. Descendendo rursus ad inferiorem columnam, est illius medius terminus $p+\frac{n^2+4n-1}{2}q$, occupans cellulam ξ K, ubi idem obtinet, ac modo diximus: Si iterum ad inferiorem lineam transversam pergamus, erit illius medius terminus $p+\frac{n^2+6n-1}{2}q$ situs in loculo σ I; & extendendo adhuc quadratum inveniretur in cellula π H quantitas $p+\frac{n^2+8n-1}{2}q$, & sic ulterius. Jam a medio totius quadrati termino, quem modo vidimus esse $p+\frac{n^2-1}{2}q$, si ad superiorem lineam transversam adscendamus, medium illius columnae terminum invenimus esse $p+\frac{n^2-2n-1}{2}q$ occupantem cellulam λ N, & ad ejus dextram positos rursus continuo quantitate q augeri, & ad sinistram minui.

nui. Si adhucdum adscendamus, medius illius lineæ terminus erit $p + \frac{n^2-4n-1}{2}q$, qui positus est in loculo κO , ubi idem obtinet, ac in præcedentibus: Ita etiam in cellula ιP habes terminum $p + \frac{n^2-6n-1}{2}q$, & sic porro: Ut adeo termini collocati in altera linea diagonali, ubi scil. dantur loculi $o I$, ξK , νL , μM , &c. itidem sint in progressionem Arithmetica, cujus excessus est nq . Circa alias lineas transversas ordo longe facilius intuendo detegi, quam verbis describi potest; quare, ne in re nullius plane usus justo prolixior sim, hoc jam data opera transmittito. Sed de summa terminorum in quavis columna nonnihil dicendum restat: Hanc vero sermone Gallico jam designavit Cl. Frenicle en son *Traité du Quarré Magique*, qui reperitur dans les *Ouvrages adoptez par l'Académie Royale des Sciences*, Tom. II. Part. 2. pag. 210. Ego autem sermone Algebraico hanc esse dico $n \times p + \frac{n^2-1}{2}q$, adeoque in toto quadrato erit summa terminorum $n^2 \times p + \frac{n^2-1}{2}q$. Nunc, ut de veritate eorum, quæ hoc Capite diximus, evidentius constare queat, quadratum hocce magicum ad quosdam particulares casus adplicaturi sumus.

E X E M P L U M I.

Ponatur $n=3$, erit quadrati magici trium dispositio sequens,

$p+3q$	$p+8q$	$p+q$
$p+2q$	$p+4q$	$p+6q$
$p+7q$	p	$p+5q$

ubi proinde summa terminorum in quavis linea est $3p+12q$, & per totum quadratum $9p+36q$. Vides etiam omnes terminos a p usque ad $p+8q$ hic reperiri, omniaque prorsus requisita adesse, & exteriores zonas, quæ in universali quadrato reperiun-

riuntur, usui jam non venire. Interim non omnes quadrati trium dispositiones hic adfunt, quum id octo modis variari queat, juxta Cl. Struyck in *syne Giffingen over de Staet van 't Menschelyck Geslacht* pag. 323. Quin nec ad id, ut quadratum dici possit magicum, præcise requiritur, ut omnes termini progressionis Arithmeticæ adfint, velut ex sequenti schemate liquet,

$\frac{2}{3}s - a$	$\frac{2}{3}s - b$	$-\frac{1}{3}s + a + b$
$-\frac{2}{3}s + 2a + b$	$\frac{1}{3}s$	$\frac{4}{3}s - 2a - b$
$s - a - b$	b	a

ubi a & b quantitates sunt ad lubitum sumptæ, s vero summa terminorum in quavis columna; sicque infinitis modis tali casu quadratum hocce variari potest.

E X E M P L U M I I.

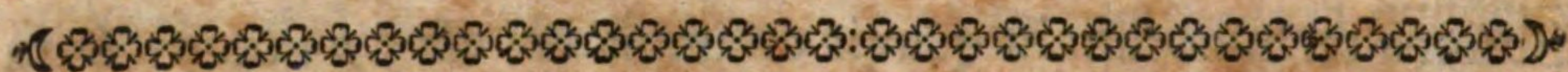
Ponatur $n = 5$, & erit quadrati magici quinque loculorum dispositio hujusmodi:

$p + 10q$	$p + 23q$	$p + 6q$	$p + 19q$	$p + 2q$
$p + 3q$	$p + 11q$	$p + 24q$	$p + 7q$	$p + 15q$
$p + 16q$	$p + 4q$	$p + 12q$	$p + 20q$	$p + 8q$
$p + 9q$	$p + 17q$	p	$p + 13q$	$p + 21q$
$p + 22q$	$p + 5q$	$p + 18q$	$p + q$	$p + 14q$

Summa

Summa terminorum in quavis columna hic est $5p + 60q$, & in toto quadrato $25p + 300q$. Cl. Frenicle autem *dict. Tract. pag. 228. & 230.* ait, in tali quadrato magico, ubi $n = 5$, & ubi numerus 13, sive generalius, ubi $p + 12q$ in medio collocatus est, dari non posse 240 permutationes; verum ea in re magnopere fallitur; quum Vir Doctissimus Wilh. La Bordus in Academia Lugduno-Batava Matheseos Lector dignissimus (cujus institutione in Geometria, Algebra, & Astronomia jam per integrum sexennium usus fui,) postquam hanc generalem quadrati magici dispositionem a me dudum inventam ipsi exhibuissem, mihi ostenderit schedas quasdam a nonnemine, qui omne vitæ tempus eam in rem impenderat, magna cum cura conscriptas, in quibus, posito numero 13 in medio totius quadrati, & præterea unitate in medio infimæ columnæ horizontalis, id ipsum 853 diversis modis repræsentaverat; quem proinde numerum si multiplicemus per 25, erunt omnes permutationes, quas quadratum illud magicum manente 13 in medio subire potest, numero 21325, qui circiter octogies & novies superat numerum 240. Neque præterea requiritur, ut præcise numerus 13 medium totius quadrati occupet; quum teste Dn. Frenicle *d. l. pag. 231. & seqq.* omnes numeri impares (nam de paribus dubitat,) in medio collocari possint; ut proinde ex hisce videamus, quam innumeris modis quadratum magicum, ubi numerus loculorum duntaxat æqualis est 5, transponi queat.





C A P U T V I.

Quadrati magici, ubi numerus loculorum est par, constructio universalis.

DEtur rursus quadratum naturale quotcunque loculorum, (quorum numerum, quem jam parem suppono, exprimo per n ,) in quo omnes termini progressionis Arithmeticae a p excessu q usque ad $p+n^2-1.q$ reperiuntur, poterit illud quadratum naturale in artificiale seu magicum transformari hac ratione:

$p+18q$	$p+n^2-21.q$	$p+n^2-22.q$	$p+n^2-23.q$	$p+n^2-24.q$	$p+25q$	$p+24q$	$p+19q$
$p+n^2-27.q$	$p+8q$	$p+n^2-12.q$	$p+n^2-11.q$	$p+n^2-14.q$	$p+17q$	$p+9q$	$p+26q$
$p+n^2-29.q$	$p+15q$	p	$p+n^2-2.q$	$p+n^2-3.q$	$p+3q$	$p+n^2-16.q$	$p+28q$
$p+n^2-32.q$	$p+n^2-17.q$	$p+n^2-5.q$	$p+5q$	$p+6q$	$p+n^2-8.q$	$p+16q$	$p+31q$
$p+30q$	$p+n^2-13.q$	$p+7q$	$p+n^2-7.q$	$p+n^2-6.q$	$p+4q$	$p+12q$	$p+n^2-31.q$
$p+29q$	$p+14q$	$p+n^2-4.q$	$p+2q$	$p+q$	$p+n^2-1.q$	$p+n^2-15.q$	$p+n^2-30.q$
$p+27q$	$p+n^2-10.q$	$p+11q$	$p+10q$	$p+13q$	$p+n^2-18.q$	$p+n^2-9.q$	$p+n^2-28.q$
$p+n^2-20.q$	$p+20q$	$p+21q$	$p+22q$	$p+23q$	$p+n^2-26.q$	$p+n^2-25.q$	$p+n^2-19.q$

Si

Si cui volupe est, quadratum hocce ulterius extendere poterit, hac tamen adhibita cautela, ut summa duorum terminorum, qui in cellulis sibimet e regione oppositis siti sunt, semper faciat $2p + \overline{n^2 - 1}.q$: Est vero quadratum illud hac ratione constructum, ut dempto uno pluribusve exterioribus cingulis quadratum superstes interius habeat conditiones requisitas, adeoque sit magicum: Cæteroquin potest & quadratum magicum ita construi, ut demptis una pluribusve zonis interius quadratum non sit magicum. Vid. Cl. Frenicle *prædict. Tract. pag. 291.* verum de hac specie, quippe quæ minus involvit elegantiae, agere propositum nostrum non sivit. In prædicto itaque casu summa terminorum in quavis columna integri quadrati est $\frac{n}{2} \times 2p + \overline{n^2 - 1}.q$, & in universum $\frac{n}{2} \times 2p + \overline{n^2 - 1}.q$, perinde atque circa quadratum impar antea vidimus: Deinde dempto extremo cingulo remanet quadratum magicum, ubi summa terminorum in quavis linea est $\frac{n-2}{2} \times 2p + \overline{n^2 - 1}.q$, & universum $\frac{n-2}{2} \times 2p + \overline{n^2 - 1}.q$: Iterum ablata zona superest quadratum, ubi termini omnes in qualibet columna additi junctim efficiunt $\frac{n-4}{2} \times 2p + \overline{n^2 - 1}.q$, & in toto quadrato $\frac{n-4}{2} \times 2p + \overline{n^2 - 1}.q$, & sic porro: Demptis itaque cingulis numero l erit summa terminorum in quacunque superstitis quadrati columna $\frac{n-2l}{2} \times 2p + \overline{n^2 - 1}.q$, & in toto quadrato $\frac{n-2l}{2} \times 2p + \overline{n^2 - 1}.q$; idque perpetuo, usque dum tale quadratum remaneat, ut ex primum dato quadrato auferantur zonæ numero $\frac{n-4}{2}$, (ubi proinde $l = \frac{n-4}{2}$) hoc est, usque dum restet quadratum, ubi cellulae in quavis columna dantur numero 4, quorum summa tum erit $2 \times 2p + \overline{n^2 - 1}.q$, & in toto quadrato $8 \times 2p + \overline{n^2 - 1}.q$; nam intimum quadratum, ubi loculi in quavis columna reperiuntur duo, (ubi proinde $l = \frac{n-2}{2}$ foret,) recipere nequit conditiones ad magicum requisitas, nisi omnes termini ponerentur æquales, quod ridiculum foret. Aggregatum igitur terminorum dempto continuo cingulo decrescit in continua ratione pro quavis columna, ut

$n : n-2 : n-4 : \&c.$ & pro toto quadrato ut $n^2 : \overline{n-2}^{(2)} : \overline{n-4}^{(2)} : \&c.$
 Et generaliter summa terminorum demptis zonis numero k est
 ad summam terminorum ablatis cingulis numero l , pro quavis
 columna, ut $n-2k$ ad $n-2l$, & pro ipso quadrato, ut $\overline{n-2k}^{(2)}$
 ad $\overline{n-2l}^{(2)}$. Nunc iterum totam hanc rem quibusdam exemplis
 illustrabimus.

E X E M P L U M I.

Minimus numerus, qui pro n substitui potest, est 4; nam
 si $n=2$, quadratum nunquam erit magicum, ut modo
 vidimus: Igitur posito $n=4$, sequenti modo artificiale quadra-
 tum repræsentabitur:

p	$p+14q$	$p+13q$	$p+3q$
$p+11q$	$p+5q$	$p+6q$	$p+8q$
$p+7q$	$p+9q$	$p+10q$	$p+4q$
$p+12q$	$p+2q$	$p+q$	$p+15q$

Summam hic vides in quavis columna $4p+30q$, & in toto
 quadrato $16p+120q$: Et sic quadratum hoc uno modo dispo-
 situm habes: At vero quam plurimæ dantur hujus tam exigui
 quadrati permutationes, quales numero 880 exhibuit Dn. Fre-
 nicle *sepius citato Opere a pag. 303. usque ad 367.* sed illum
 multum adhuc a vero aberasse ait Nic. Struyck *antea memorato*
Tractatu pag. 323. qui id 5760 modis transponi posse scribit,
 quod utrum recte sese habeat nec ne, examinandum iis relin-
 quo, qui otio abuti volunt; nec enim mihi unquam persuaderi
 patiar,

patiar, quod Adamus Adamandus Kochanskus in *Act. Erud. Lips. A.* 1686. pag. 392. scribit, multiplicem hanc collocatio-
num varietatem intra horæ quadrantem addisci posse.

E X E M P L U M I I.

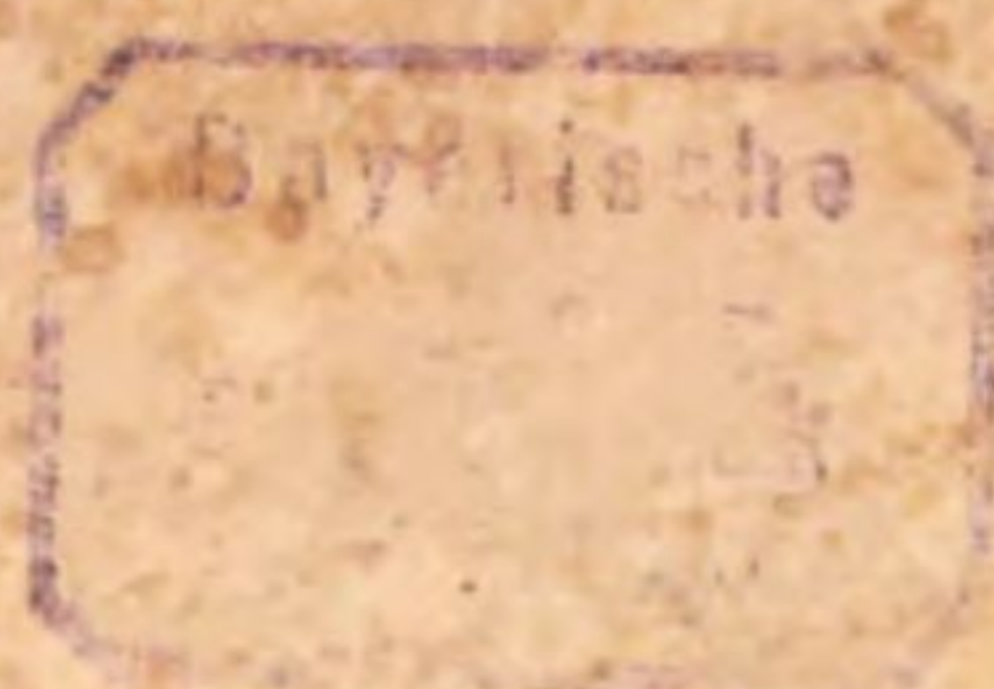
POnatur $n=6$, & erit sequens quadrati magici dispositio:

$p+8q$	$p+24q$	$p+25q$	$p+22q$	$p+17q$	$p+9q$
$p+15q$	p	$p+34q$	$p+33q$	$p+3q$	$p+20q$
$p+19q$	$p+31q$	$p+5q$	$p+6q$	$p+28q$	$p+16q$
$p+23q$	$p+7q$	$p+29q$	$p+30q$	$p+4q$	$p+12q$
$p+14q$	$p+32q$	$p+2q$	$p+q$	$p+35q$	$p+21q$
$p+26q$	$p+11q$	$p+10q$	$p+13q$	$p+18q$	$p+27q$

Summa terminorum in cellulis cujuslibet lineæ hic est $6p+105q$,
& in toto quadrato $36p+630q$. Dempta zona superest qua-
dratum, ubi summa terminorum in quavis columna est $4p+70q$,
& in univcrsum $16p+280q$.

F I N I S.

SPHAL.



S P H A L M A T A.

Pag. 12. lin. ult. *post* Incrementis *adde*: vel decrementis.

Pag. 28. lin. 3. pro $x^{\frac{1}{2}} y z$ lege: $x y z^{\frac{1}{2}}$.

Pag. 36. Exempl. 5. in princ. *pro* quantitatis *rescribe*: quantitas.

Pag. 37. in med. *pro* Aud *reponere*: Aut.

Pag. 40. in ipso fin. *pro* $x^{-2} - \&c.$ substitue: $x^{-3} - \&c.$

Pag. 58. lin. 16. *babebis* *corrigere*: habebis.

Pag. 70. lin. 4. *quod emenda*: quoad.

Pag. 80. lin. 9. *deleatur* *hæcce* *interpunctionis* *nota* (;) *post* *verbum* *datam*.

H A G Æ - C O M I T U M,

Ex Typographia A N T O N I I D E G R O O T.

M D C C X L I I.

Meerman, Gerard,

Specimen calculi Fluxionalis

Lugdunum Batavorum 1742

4 Math.p. 229

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